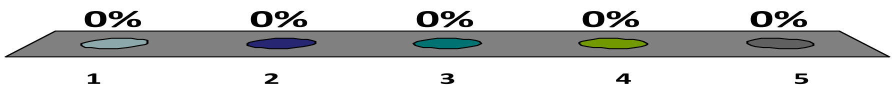
The interval between bus arrivals is $\sim U(10\text{mn}, 20\text{mn})$.

1. There are 2 buses in average per hour
2. There are 4 buses in average per hour
3. None of the above
4. I don't know

The validity of the formula obtained in the previous question requires that …

1. The interarrivals are iid
2. The bus arrival process is Poisson
3. The bus arrival process is stationary
4. None of the above
5. I don't know



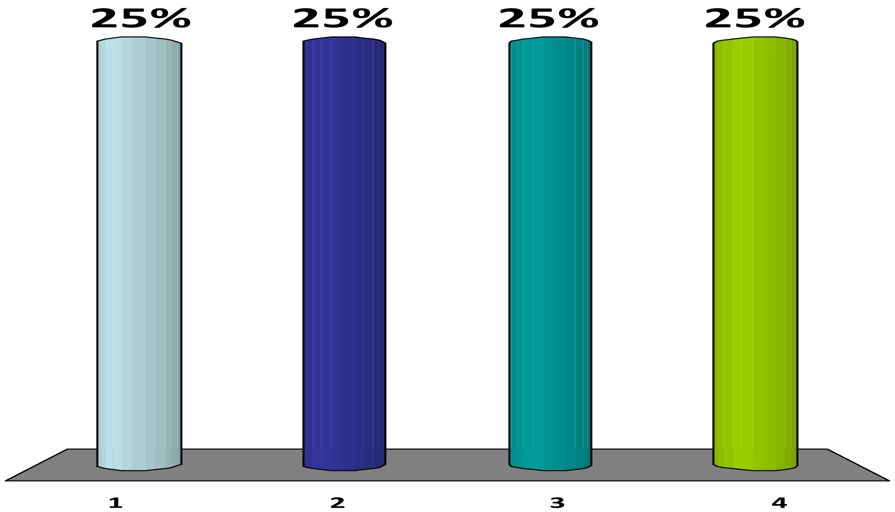
For the random waypoint model, the location of the next waypoint is…

1. Uniformly distributed
2. Not uniformly distributed
3. It depends on the viewpoint
4. I don't know

BorduRail claims that only 5% of train arrivals are late

BorduKonsum claims that 20% of train users suffer from late train arrivals

1. At least one of them lies
2. Late trains have $\approx 1.15 \times$ more passengers than the average train
3. Late trains have $\approx 4 \times$ more passengers than the average train
4. I don't know



Solution

N arrival events

$D_n = 1 \iff$ arrival n is late

$D_n = 0 \iff$ arrival n is on time

$P_n =$ # passengers leaving at n^{th} arrival event.

BordnRail: $\bar{D} = \frac{1}{N} \sum_{n=1}^{N} D_n$

BordnKonsum: $D^* = \frac{1}{\sum P_n} \sum_{n=1}^{N} \sum_{p=1}^{P_n} D_n = \frac{1}{N\bar{P}} \sum_{n:D_n=1} P_n$

with $\bar{P} = \frac{1}{N} \sum_{n=1}^{N} P_n$

$\bar{P}_1 = \frac{1}{N_{late}} \sum_{n:D_n=1} P_n$

$N_{late} = N \cdot \bar{D} = \sum_{n=1}^{N} D_n$

$\Rightarrow D^* = \frac{1}{N\bar{P}} \bar{P}_1 N \bar{D}$

$$D^* = \bar{D} \times \frac{\bar{P}_1}{\bar{P}}$$

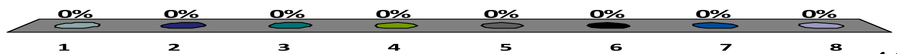
$\bar{P}_1$: av. # passengers in train when late

$\bar{P}$: av # passengers at an arrival

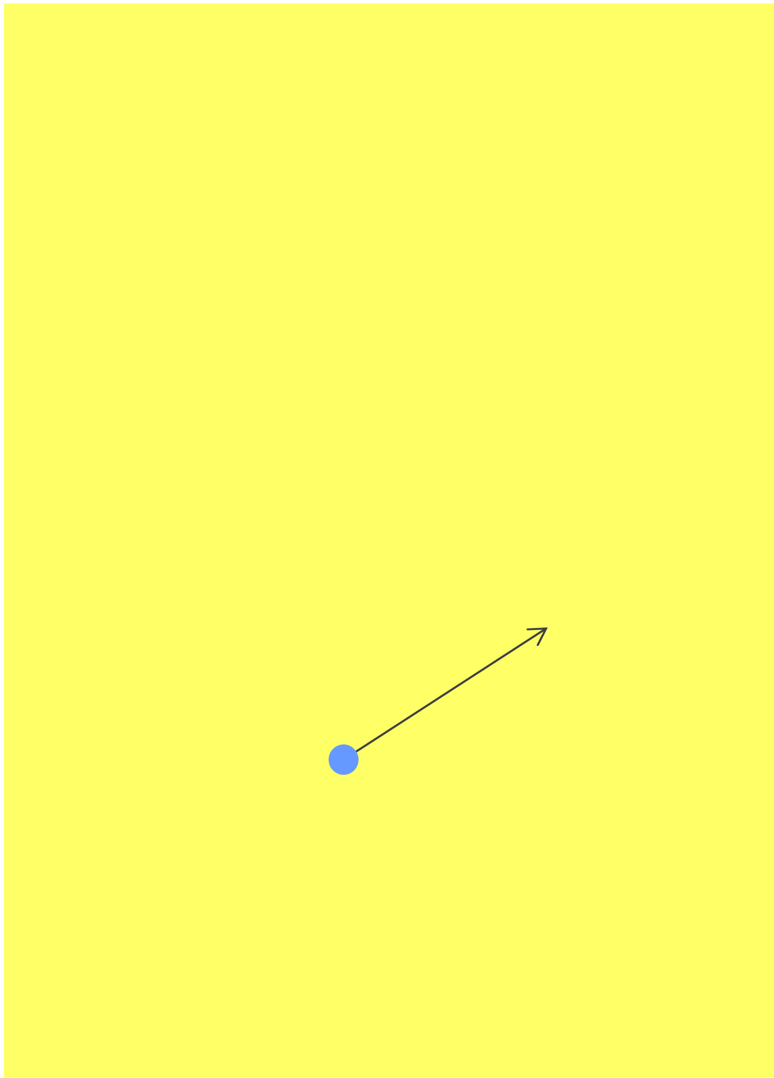
A sensors senses events; the time between events (sensing interval) is $\sim N(\mu, \sigma^2)$.

A technician comes and checks the current sensing interval. In average, he will find…

1. μ
2. $\mu + \sigma^2$
3. $\mu\,(1 + \sigma^2)$
4. $\mu\left(1 + \frac{\sigma^2}{\mu^2}\right)$
5. $\frac{1}{\mu}\left(1 + \frac{\sigma^2}{\mu^2}\right)$
6. $\frac{1}{\mu}\left(1 + \frac{\sigma^2}{\mu}\right)$
7. None of the above
8. I don't know

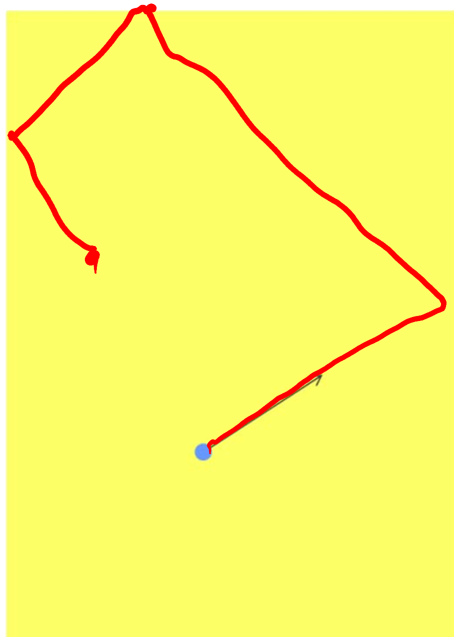


A mobile moves as follows
- pick a random direction uniformly in $[0, 2\pi]$
- pick a random trip duration T ~Pareto(p)
- go in this direction for duration T at constant speed ; if needed reflect at the boundary.

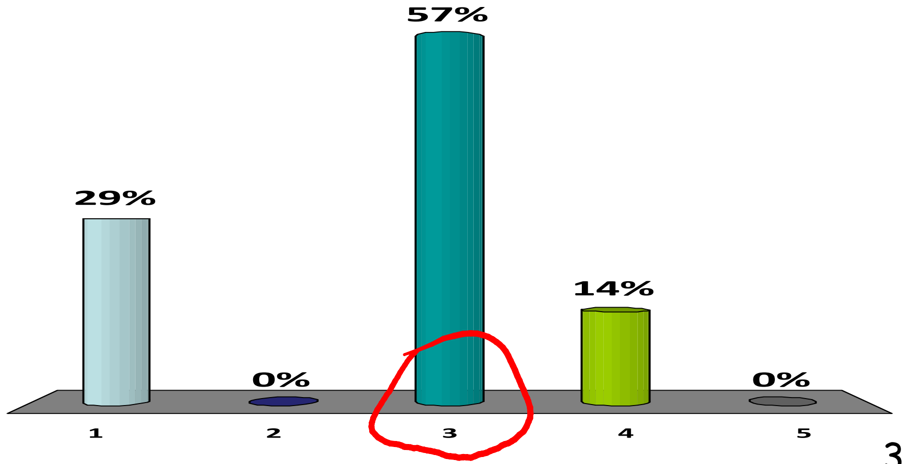


A mobile moves as follows
- pick a random direction uniformly in $[0, 2\pi]$
- pick a random trip duration T ~Pareto(p)
- go in this direction for duration T and if needed reflect at the boundary.

Does this model have a stationary regime ?



1. Yes
2. No
3. Only if $p > 1$
4. Only if $p > 2$
5. I don't know



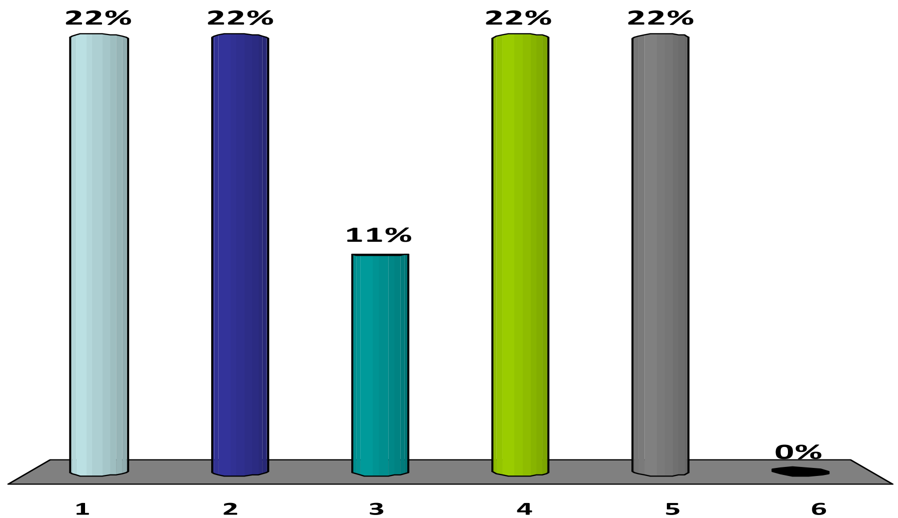
Consider the random waypoint model, where the distribution of the speed drawn at a random waypoint has a density $f(v)$ over the interval $[0, vmax]$.

Is it possible to find $f()$ such that

(1) the model has a stationary regime and YES

(2) the time stationary distribution of speed is uniform over $[0, vmax]$? YES

1. Yes, and $f()$ is uniform
2. Yes, and $f()$ is piecewise linear
3. Yes, and $f()$ is piecewise quadratic
4. Yes, but $f()$ is none of the above
5. No
6. I don't know

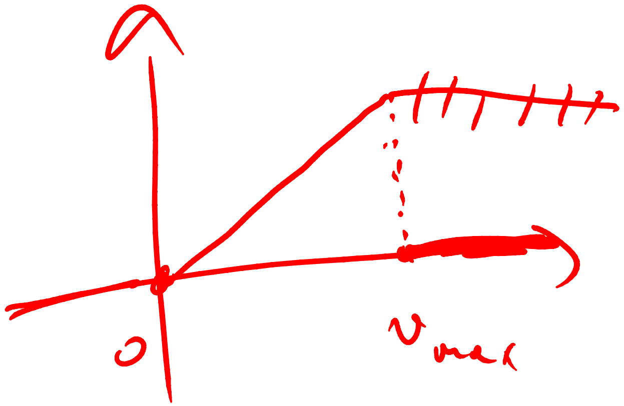


Solution

$$f_{V_1(t)}(v) = \frac{k}{v} f(v)$$

$\downarrow$ uniform

$$= \frac{1}{v_{max}} 1_{0 \le v \le v_{max}}$$

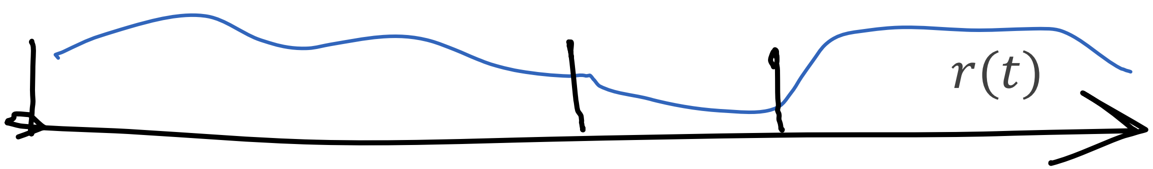


$$f(v) = \frac{1}{k\, v_{max}} v \; 1_{0 \le v \le v_{max}}$$

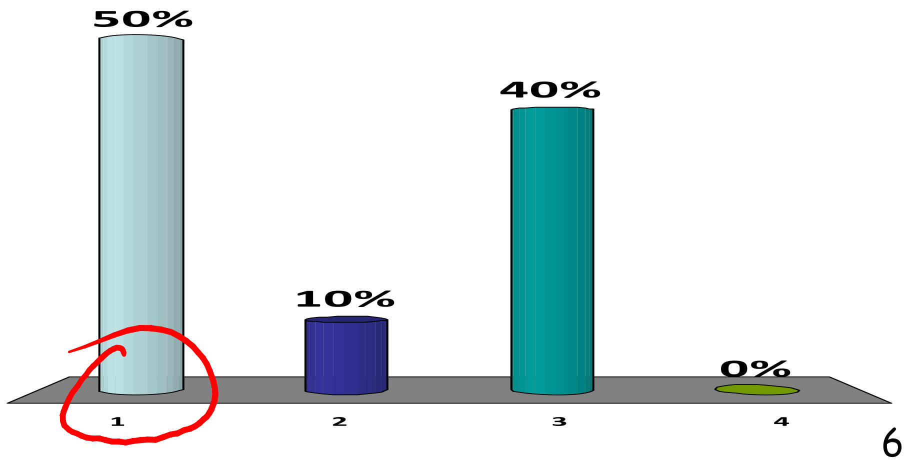
$$\Rightarrow \mathbb{E}^0\left(\frac{1}{V_0}\right) < \infty \Rightarrow \text{STATIONARY}$$

A wireless channel has a fluctuating rate $r(t)$ and operates in rounds. The average duration of a round is $\bar{T}$. The average amount of data transferred in one round is $\bar{B}$.

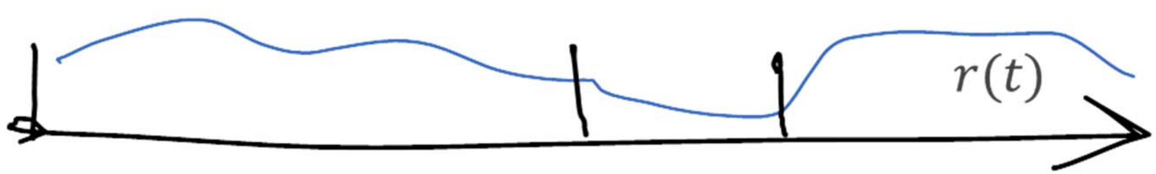
We sample the channel using the instants of a Poisson process. The average rate sampled is …



1. $\frac{\bar{B}}{\bar{T}}$
2. $\frac{\bar{T}}{\bar{B}}$
3. None of the above, it depends on the higher moments of the average round duration
4. I don't know



Solution



PASTA $\Rightarrow \bar{r} = \mathbb{E}(r(t))$

1) Large Time Heuristic: TTOT $\qquad \bar{r} = \frac{1}{T_{TOT}} \int_0^{T_{TOT}} r(s)\,ds$

$= \frac{1}{T_{TOT}}$ Amount of data transmitted $= \frac{1}{T_{TOT}} N_{TOT}\, \bar{B} = \frac{\bar{B}}{\bar{T}}$

NTOT Rounds

2) $\mathbb{E}(r(t)) = \lambda\, \mathbb{E}^0\left(\int_{T_0}^{T_1} r(s)\,ds \right) = \frac{1}{\bar{T}} \bar{B}$

Exercise:

We measure the distribution of flows transferred from a web server. We find that the distribution of the size in packets of an arbitrary flow is Pareto with $p > 1$. What is the probability that, for an arbitrary packet, it belongs to a flow of length x ?

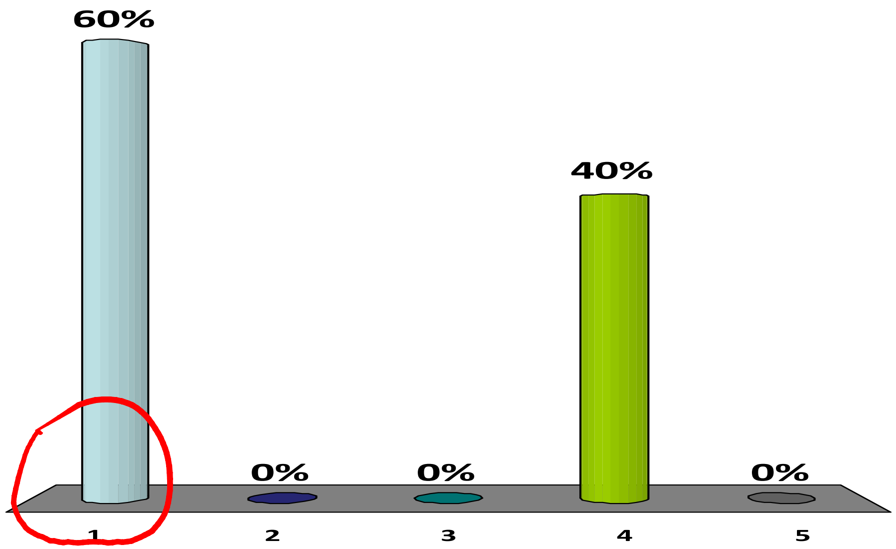
PDF of standard Pareto $f(x) = \frac{p}{x^{p+1}} 1_{x \geq 1}$

Exercise:

We measure the distribution of flows transferred from a web server. We find that the distribution of the size in packets of an arbitrary flow is Pareto. What is the probability that, for an arbitrary packet, it belongs to a flow of length x ?

The distribution is …

1. Pareto
2. Normal
3. Exponential
4. None of the above
5. I don't know



Solution

$$f_P(x) = \lambda x f_F(x)$$

$$f_F(x) = \frac{p}{x^{p+1}} 1_{x \geq 1}$$

$$\Rightarrow f_P(x) = \frac{\lambda p}{x^{p+1}} \cdot x = \frac{\lambda p}{x^p} 1_{x \geq 1}$$

$\lambda p = p - 1$ necessarily, ie $\lambda = 1 - \frac{1}{p}$

Flow sizes seen by packets is Pareto $(p-1)$

→ What if $p \leq 1$?

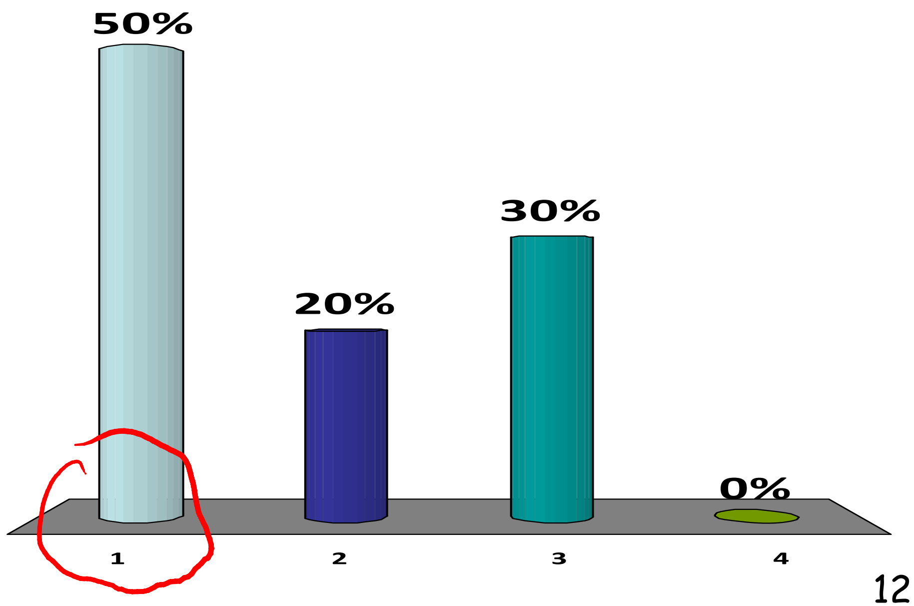
The distribution of flow sizes seen by packets is heavy tailed. Therefore the distribution of flow sizes is …

1. Also heavy tailed
2. Never heavy tailed
3. It depends
4. I don’t know

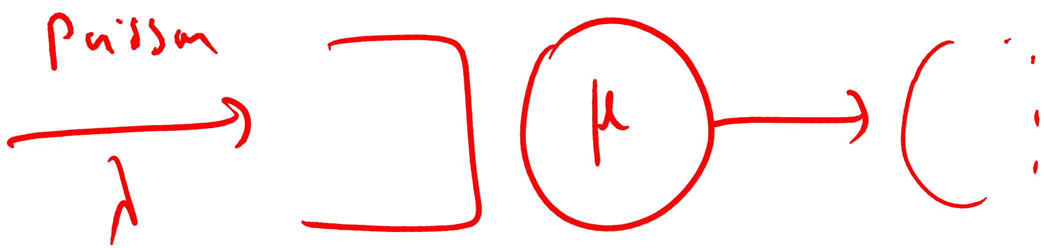
Consider an M/M/1 queue with $\rho < 1$ and consider the point process of departures.

This point process satisfies the assumptions of the PASTA theorem

1. No
2. Yes
3. It depends on the parameters of the queueing system
4. I don’t know



Solution



N(t) is Markov

if $N(t) \geq 1$ proba of a departure in $[t, t+dt] = \mu\, dt + o(dt)$

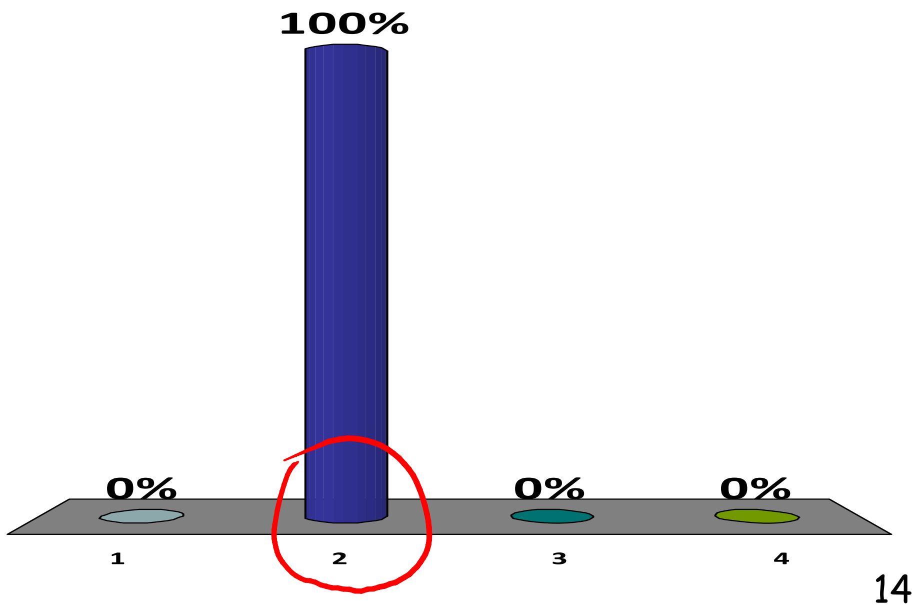
else $(N(t) == 0)$ proba $= 0$

$\Rightarrow$ NOT PASTA-able

Consider an M/M/1 queue with $\rho < 1$ and consider the point process of arrivals.

This point process satisfies the assumptions of the PASTA theorem

1. No
2. Yes
3. It depends on the parameters of the queueing system
4. I don't know



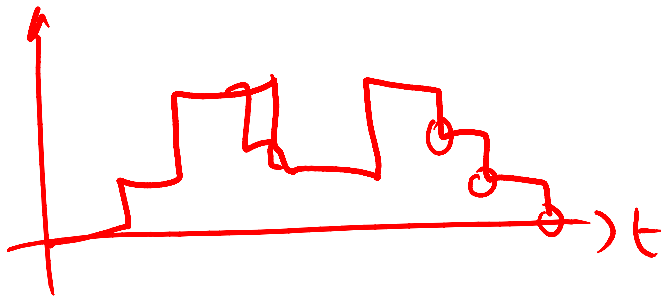
Consider an M/M/1 queue with $\rho < 1$ and consider the point process of departures.

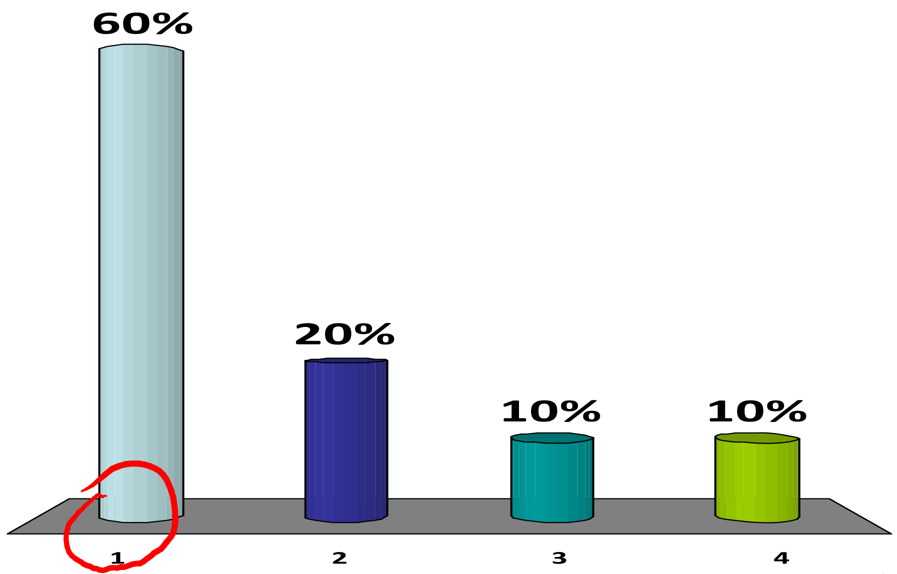
never

$N = 0$

The distribution of state just before a departure is the stationary distribution.

1. No
2. Yes
3. It depends on the parameters of the queueing system
4. I don't know

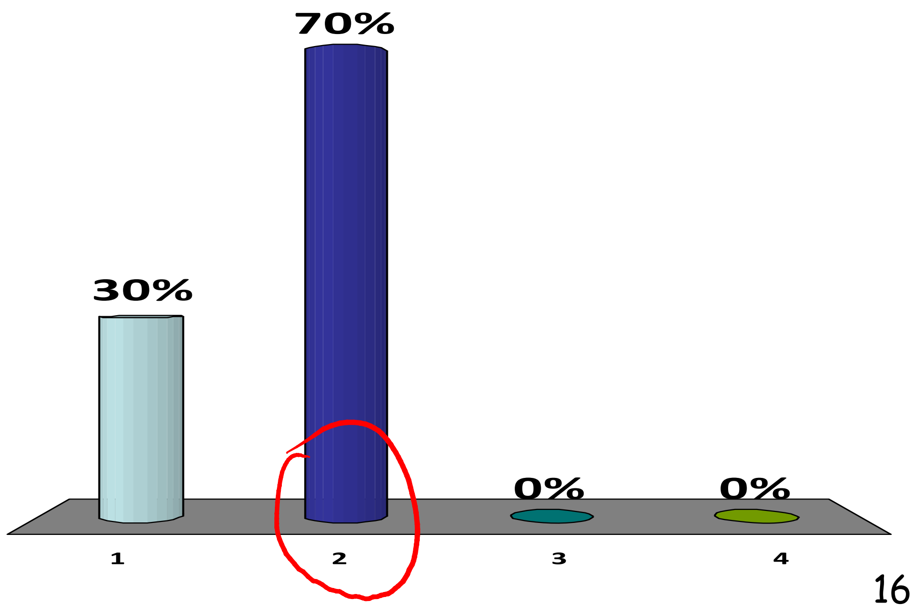




Consider an M/M/1 queue with $\rho < 1$ and consider the point process of departures.

The distribution of state just after a departure is the stationary distribution.

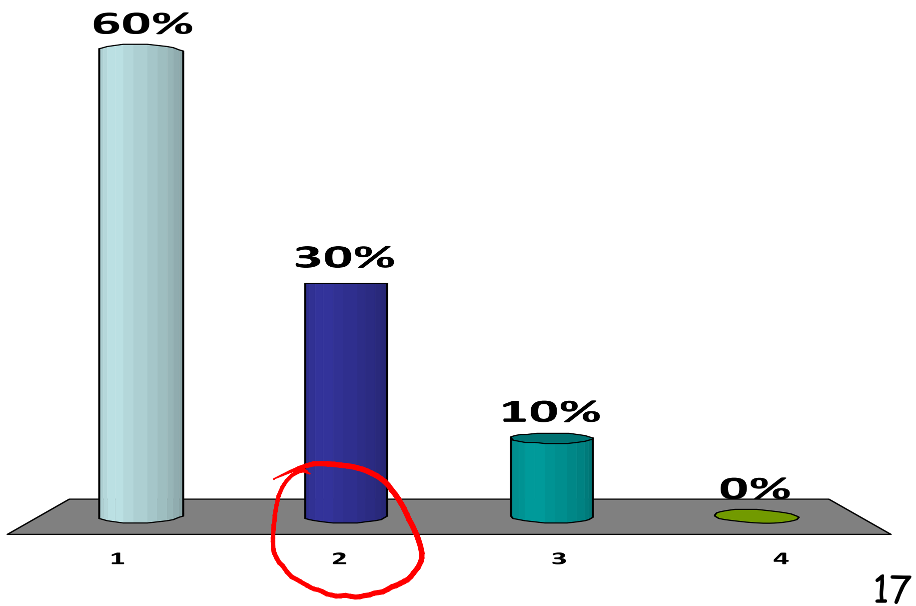
1. No
2. Yes
3. It depends on the parameters of the queueing system
4. I don't know



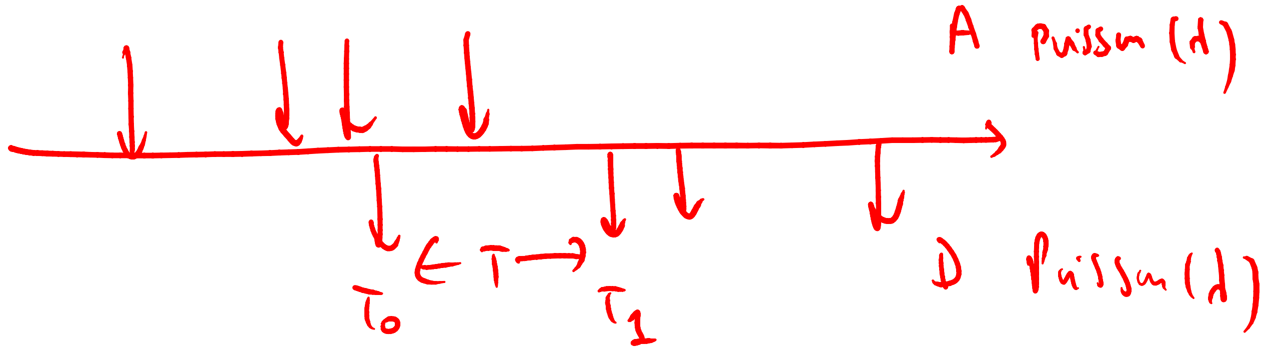
Consider an M/M/1 queue with $\rho < 1$ and consider the point process of departures.

This point process is a Poisson Process

1. No
2. Yes
3. It depends on the parameters of the queueing system
4. I don't know



Solution



M/M/1

$$E\left(e^{-sT(t)} \mid N(t)=n\right) = f_n(s) \qquad T(t) = \text{time until next departure}$$

$$\begin{cases} n \geq 1: & f_n(s) = \dfrac{\mu}{\mu+s} \quad \to \rho \\ n = 0 & f_0(s) = \dfrac{\lambda}{\lambda+s} \cdot \dfrac{\mu}{\mu+s} \quad \to 1-\rho \end{cases}$$

$$E^0\left(e^{-sT_1}\right) = (1-\rho)\frac{\lambda}{\lambda+s}\frac{\mu}{\mu+s} + \rho\frac{\mu}{\mu+s} = \frac{\mu}{\mu+s}\left((1-\rho)\frac{\lambda}{\lambda+s} + \rho\right)$$

$$= \frac{\mu}{\mu+s}\,\frac{(\mu+s)\lambda}{\mu(\lambda+s)} = \frac{\lambda}{\lambda+s}$$

$$\int_0^\infty e^{-sx} e^{-\mu x} \mu \, dx = \frac{\mu}{\mu+s}$$

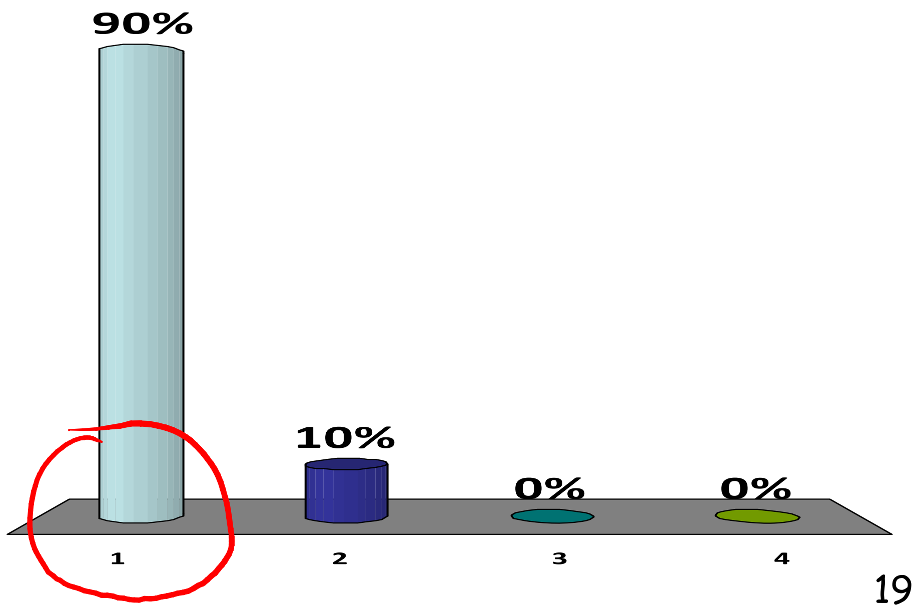
In the M/M/1/K queue, the probability that an arriving packet is discarded is …

M/M/1/K QUEUE

$$\begin{cases} \mathbb{P}(N=k) = \eta(1-\rho)\rho^k 1_{\{0\leq k \leq K\}} \\ \eta = \frac{1}{1-\rho^{K+1}} \end{cases}$$

1. $P(N=K)$
2. Is not equal to $P(N=K)$
3. It depends on the parameters
4. I don't know

PASTA



Solution

M/M/1/K QUEUE Stability is for any ρ.

$$\begin{cases} \mathbb{P}(N = k) = \eta(1-\rho)\rho^k 1_{\{0 \le k \le K\}} \\ \eta = \frac{1}{1-\rho^{K+1}} \\ \mathbb{P}^0(\text{ arriving customer is discarded }) = \mathbb{P}(N = K) \end{cases}$$