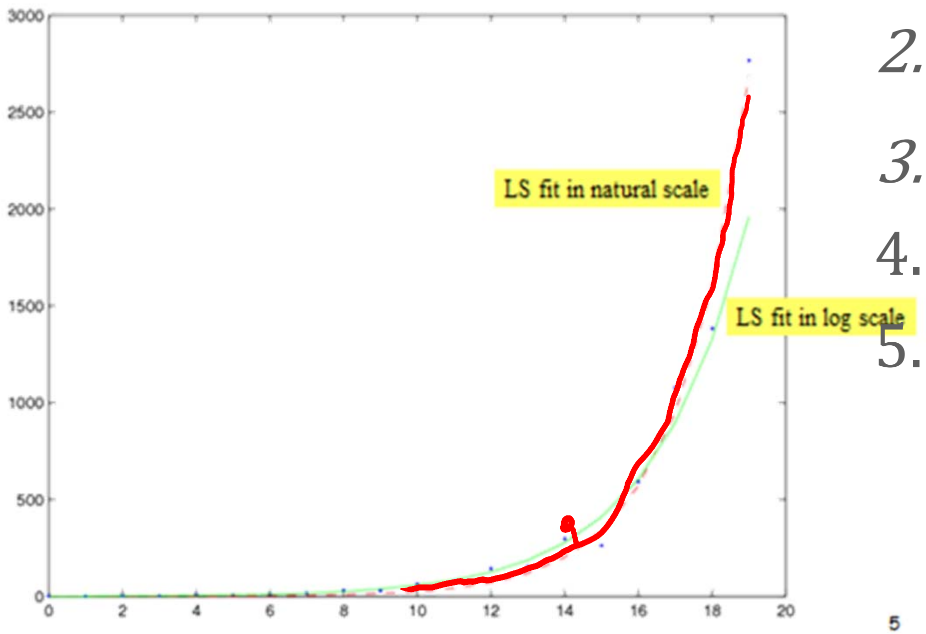
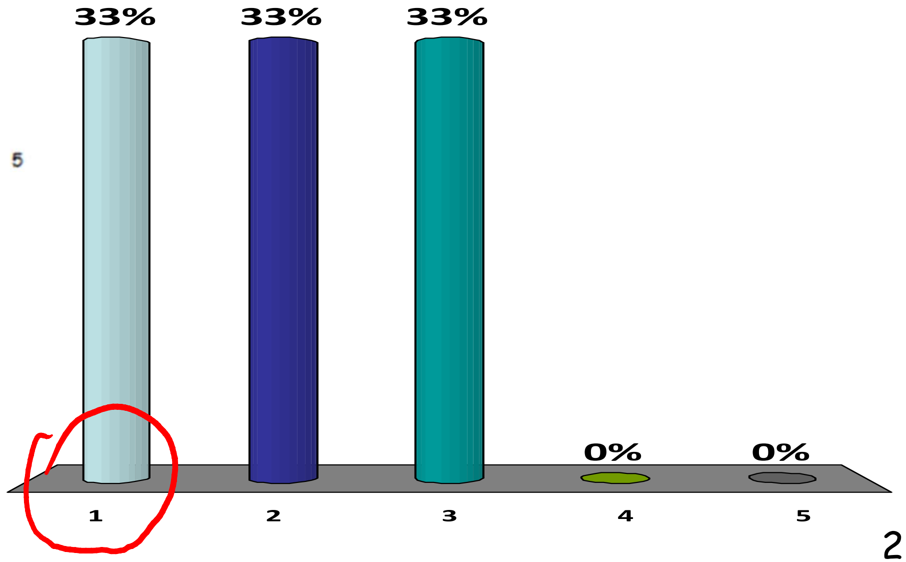
The residuals for the red estimation are …

$y_i = a e^{\alpha t_i}$

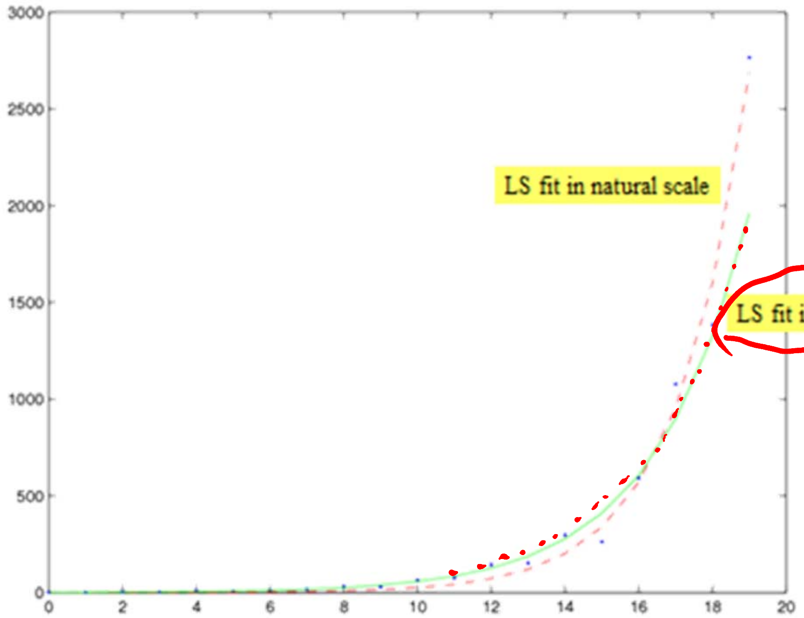
$a, \alpha \to$ fitted



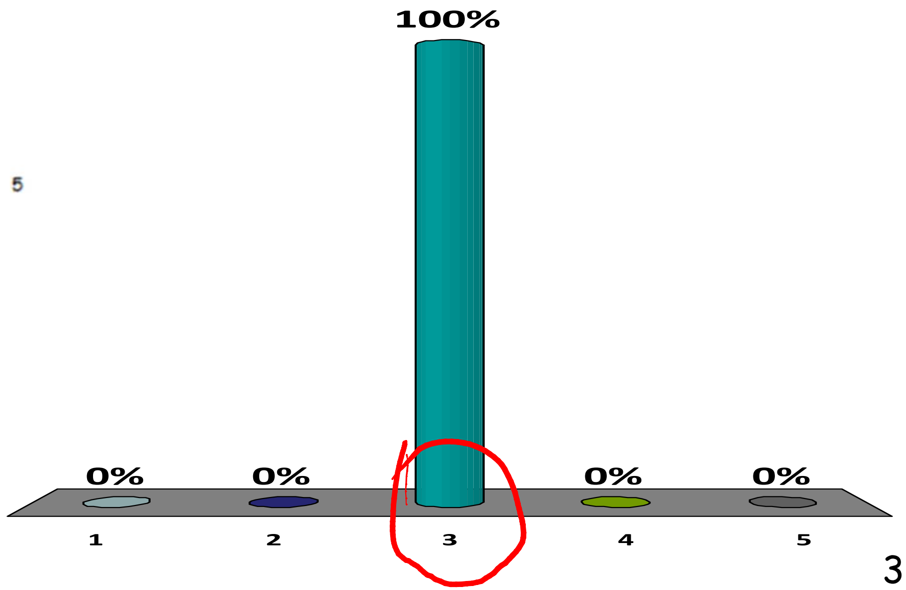
1. $e_i = y_i - a\,e^{\alpha t_i}$
2. $e_i = \dfrac{y_i - a\,e^{\alpha t_i}}{a\,e^{\alpha t_i}}$
3. $e_i = \log y_i - \log a - \alpha\,t_i$
4. None of the above
5. I don't know



The residuals for the green estimation are …

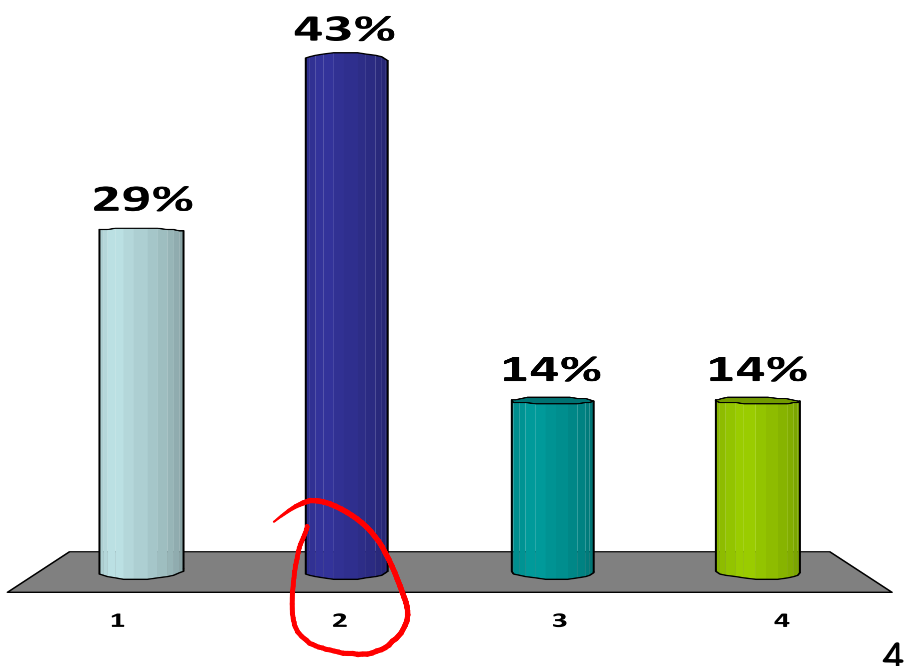


1. $e_i = y_i - a\, e^{\alpha t_i}$
2. $e_i = \dfrac{y_i - a\, e^{\alpha t_i}}{a\, e^{\alpha t_i}}$
3. $e_i = \log y_i - \log a - \alpha\, t_i$
4. None of the above
5. I don't know



If the error terms are not homoscedastic, it is better to …

1. Use ℓ^1 minimization rather than ℓ^2
2. Rescale to make the error term homoscedastic
3. Use ℓ^2 minimization rather than ℓ^1
4. I do not know

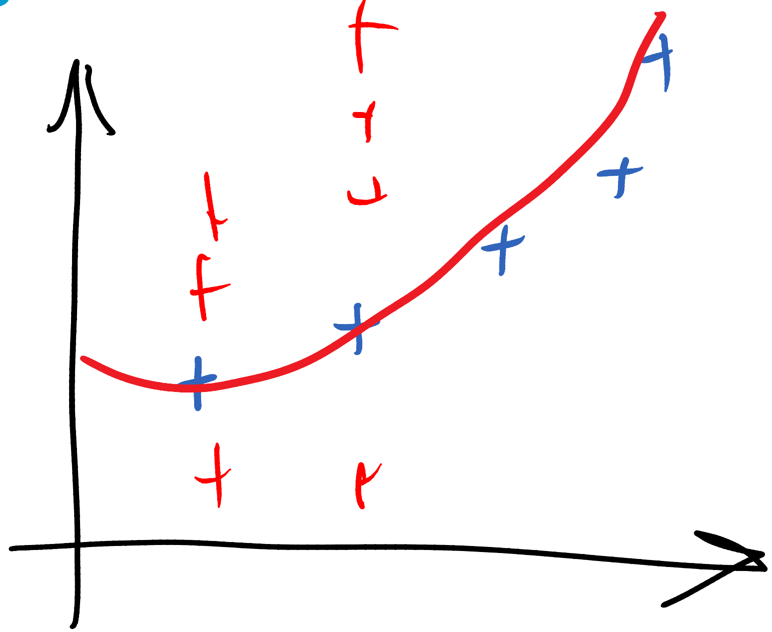


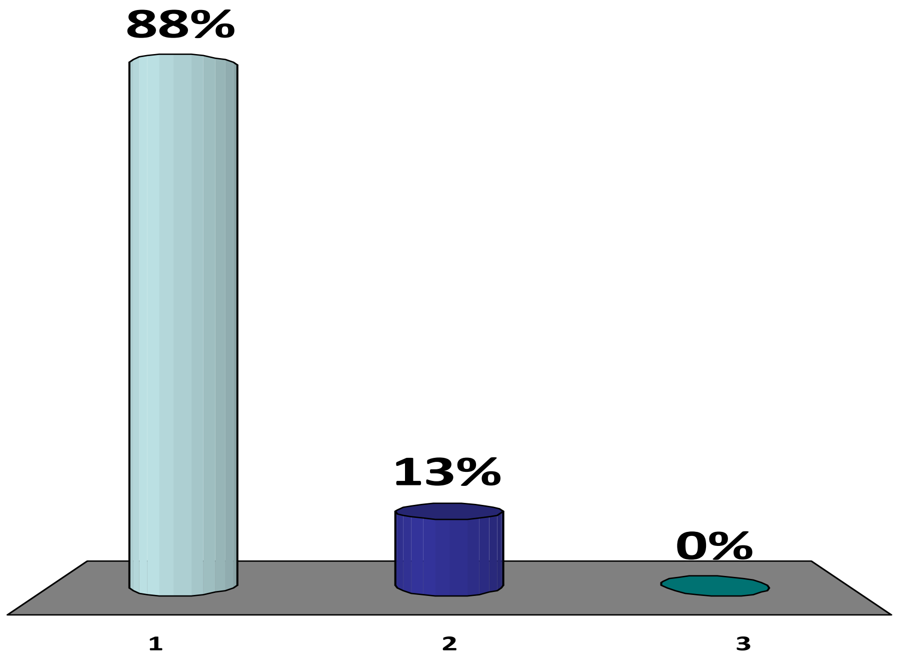
We want to fit a data set y_i to a polynomial of degree 2:

$$y_i = at_i^2 + bt_i + c.$$

Is this a linear regression model ?

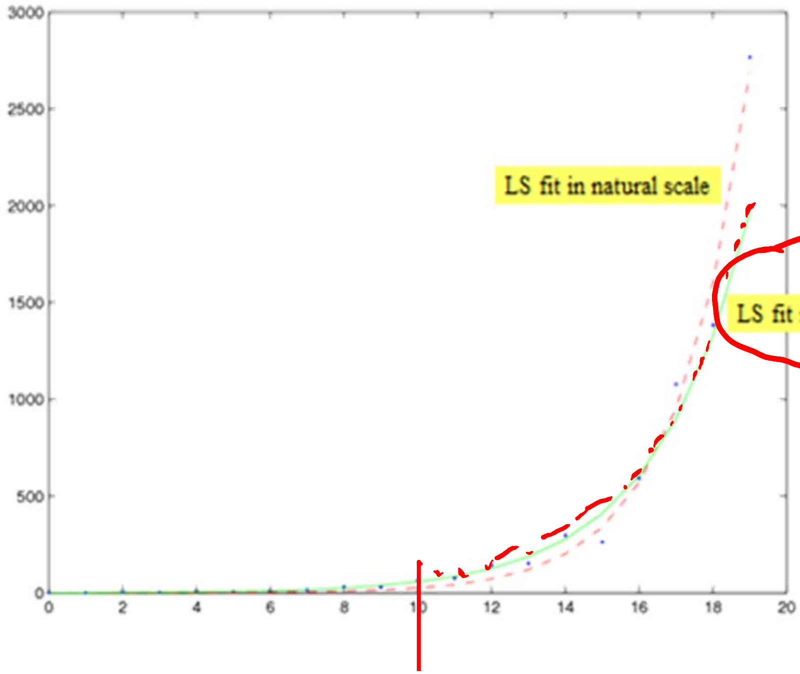
1. Yes
2. No
3. I don't know

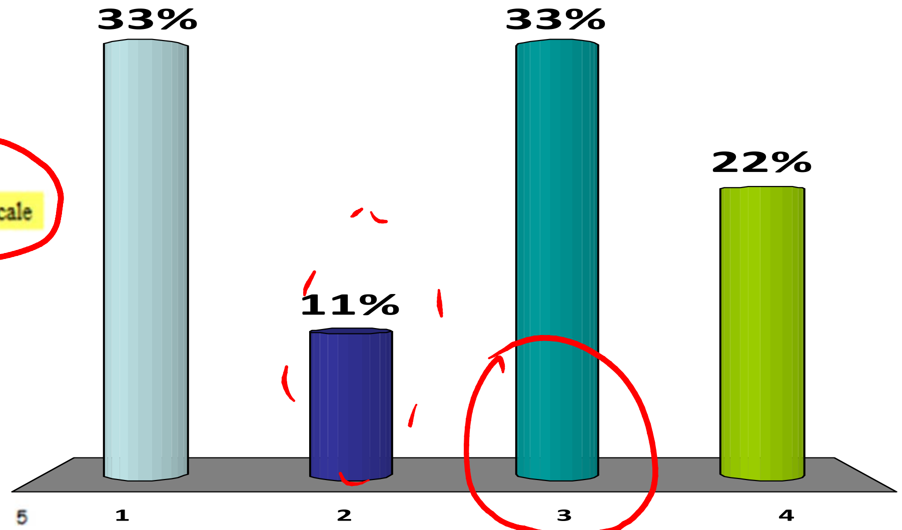




The green estimation corresponds to assuming that the error terms (blue dot – green curve) are iid lognormal.

1. True
2. Almost true
3. False
4. I don't know





Solution

$$\log y_i = \overbrace{\log f_i(\beta)}^{\log a + \alpha t_i} + \varepsilon_i$$

$$\varepsilon_i \sim \text{iid } N(0, \sigma^2)$$

$$y_i = f_i(\beta)\, e^{\varepsilon_i}$$

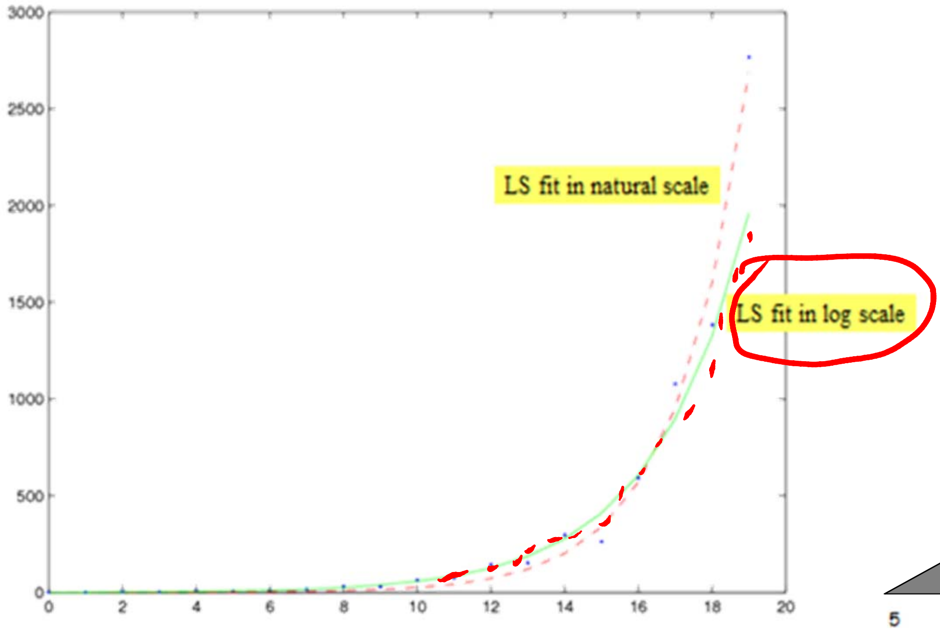
error terms: $y_i - f_i(\beta) = $

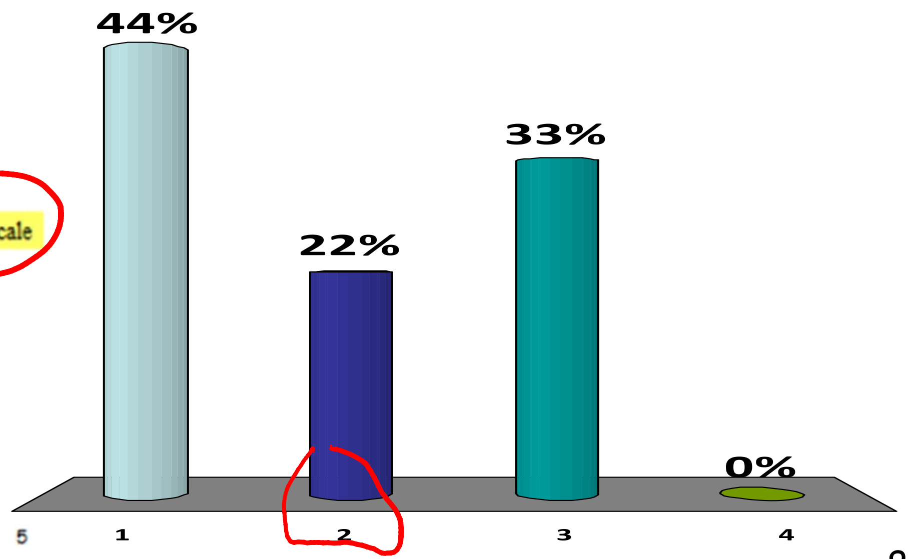
$$f_i(\beta)\underbrace{(e^{\varepsilon_i} - 1)}_{\text{lognormal (shifted)}}$$

not iid

The green estimation corresponds to assuming that the *relative* error terms (blue dot – green curve) are iid normal.

1. True
2. Almost true
3. False
4. I don't know





Solution

relative errors: $\frac{y_i - f_i(\beta)}{f_i(\beta)} \overset{\text{def}}{=} r_i$

$$r_i = \frac{f_i(\beta)\, e^{\varepsilon_i} - f_i(\beta)}{f_i(\beta)} = e^{\varepsilon_i} - 1$$

if σ^2 small, ε_i small and

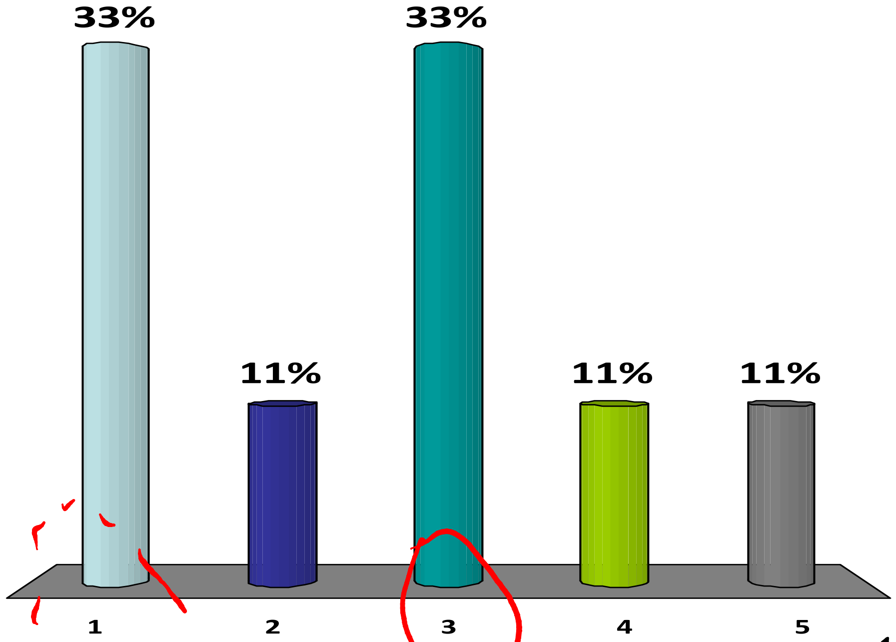
$$e^{\varepsilon_i} - 1 \simeq 1 + \varepsilon_i - 1 = \varepsilon_i$$

iid, almost normal

The X matrix for the model $y_i = at_i^2 + bt_i + c$ has full rank.

1. Yes
2. No
3. It depends on the t_i's
4. It depends on the y_i's
5. I don't know

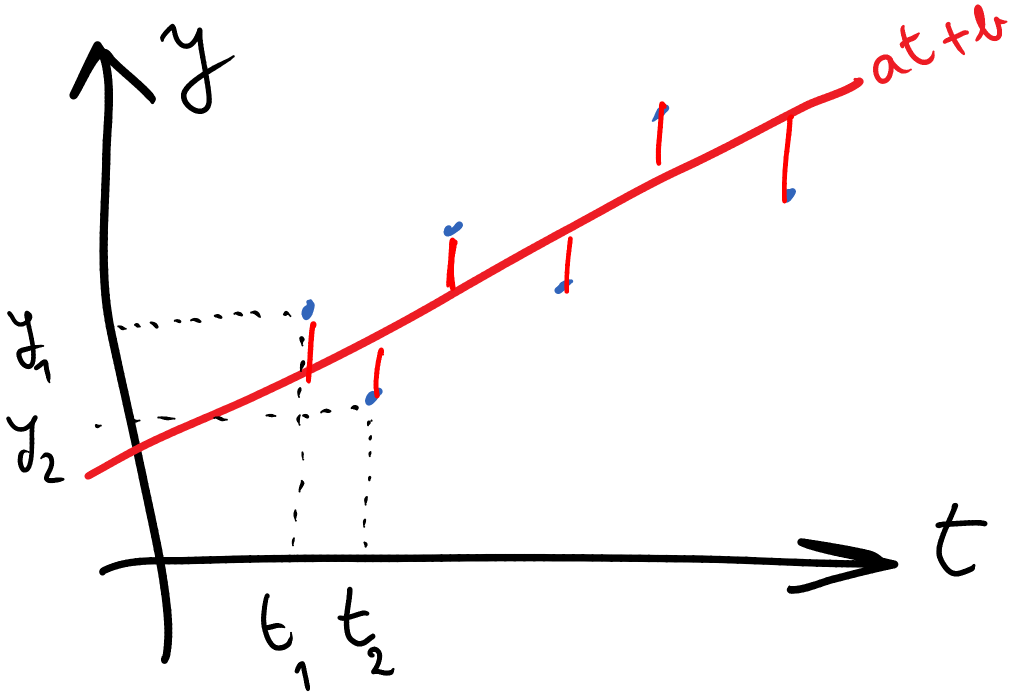
$$\begin{pmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ & \cdots & \\ t_n^2 & t_n & 1 \end{pmatrix}$$

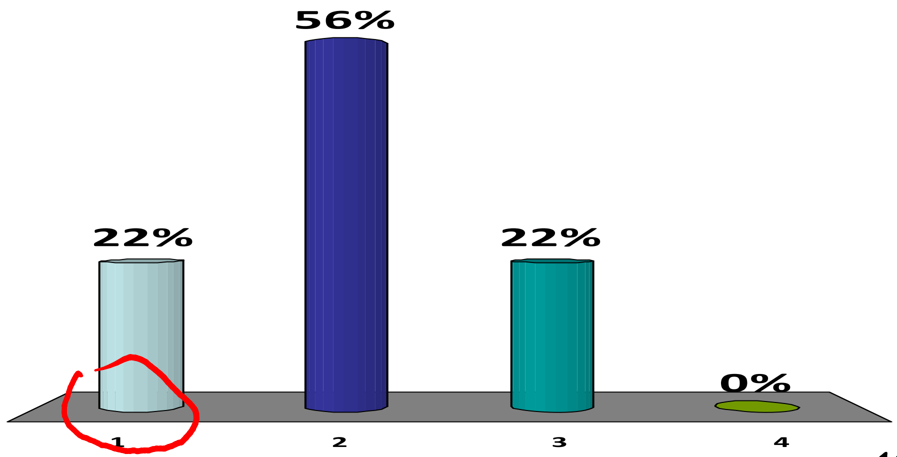


We fit the model
$$y_i = at_i + b$$
using least squares.

The obtained line is such that the vertical average distance from the points to the line is 0.

1. True
2. False
3. It depends on the data
4. I don't know





Solution $(\hat{a}, \hat{b})$ = fitted parameters.

$(\hat{a}, \hat{b})$ minimizes $\sum_i [y_i - (at_i + b)]^2$

Let $y_i - \hat{a}t_i \stackrel{\text{def}}{=} x_i$

$\hat{b}$ minimizes $\sum_i [y_i - (\hat{a}t_i + b)]^2$

$\hat{b}$ minimizes $\sum_i (x_i - b)^2$

thus $\hat{b}$ = mean of $x_i = \bar{x}$

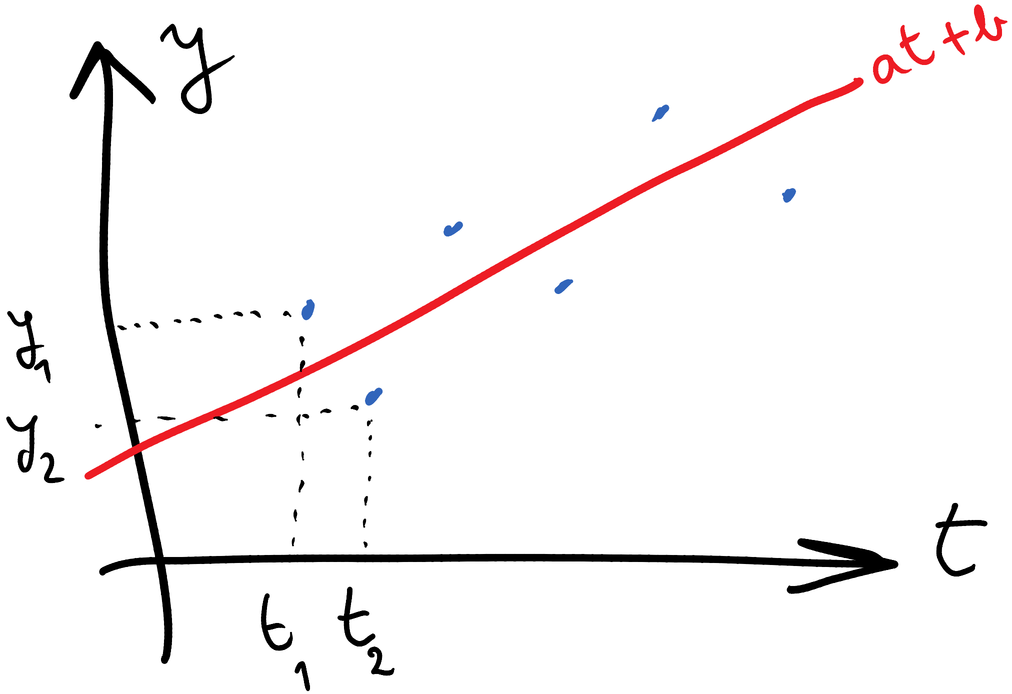
thus average of $y_i - at_i - b$ is 0

We fit the model

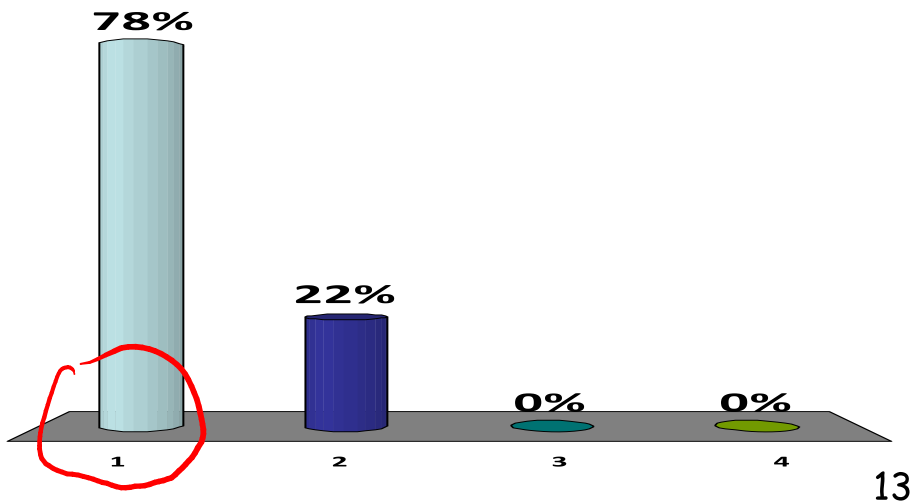
$$\boldsymbol{y_i = at_i + b}$$

using ℓ^1norm minimization.

The obtained line leaves an equal number of points on each side



1. True
2. False
3. It depends on the data
4. I don't know



Solution $(\hat{a}, \hat{b})$ = fitted parameters.

$(\hat{a}, \hat{b})$ minimizes $\sum_i |y_i - (at_i + b)|$

Let $y_i - \hat{a}t_i \stackrel{\text{def}}{=} x_i$

$\hat{b}$ minimizes $\sum_i |y_i - (\hat{a}t_i + b)|$

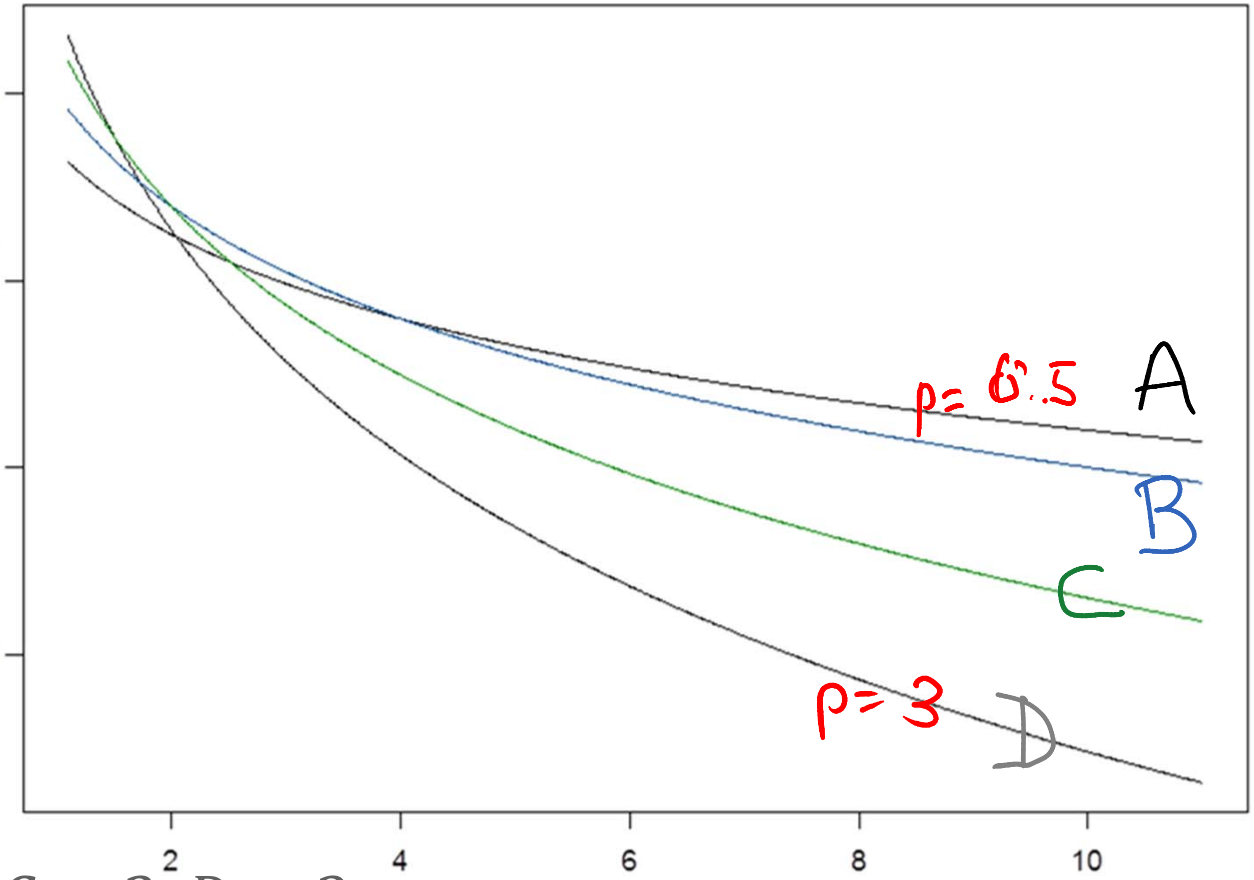
$\hat{b}$ minimizes $\sum_i |x_i - b|$

thus $\hat{b}$ = median of x_i

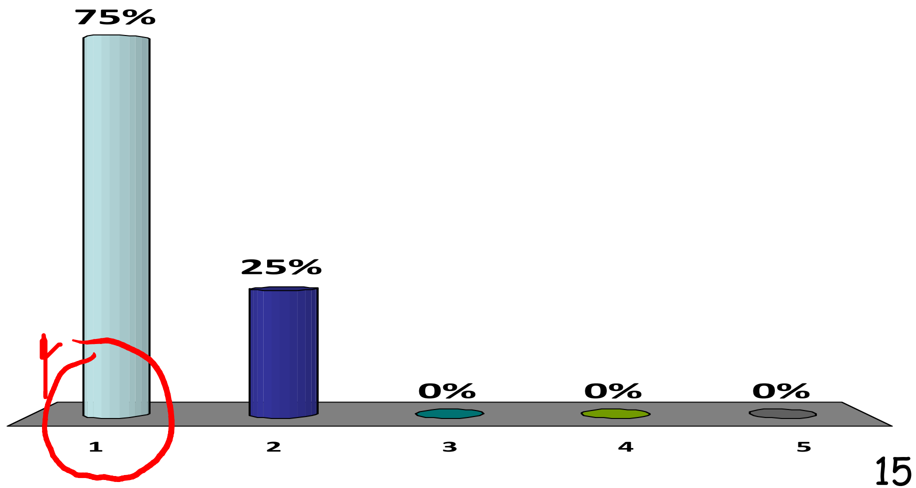
equal number of points on each side

Find the parameter p for each of these standard Pareto PDFs

$f(x) = \frac{p}{x^{p+1}} \mathbb{1}_{x \geq 1}$

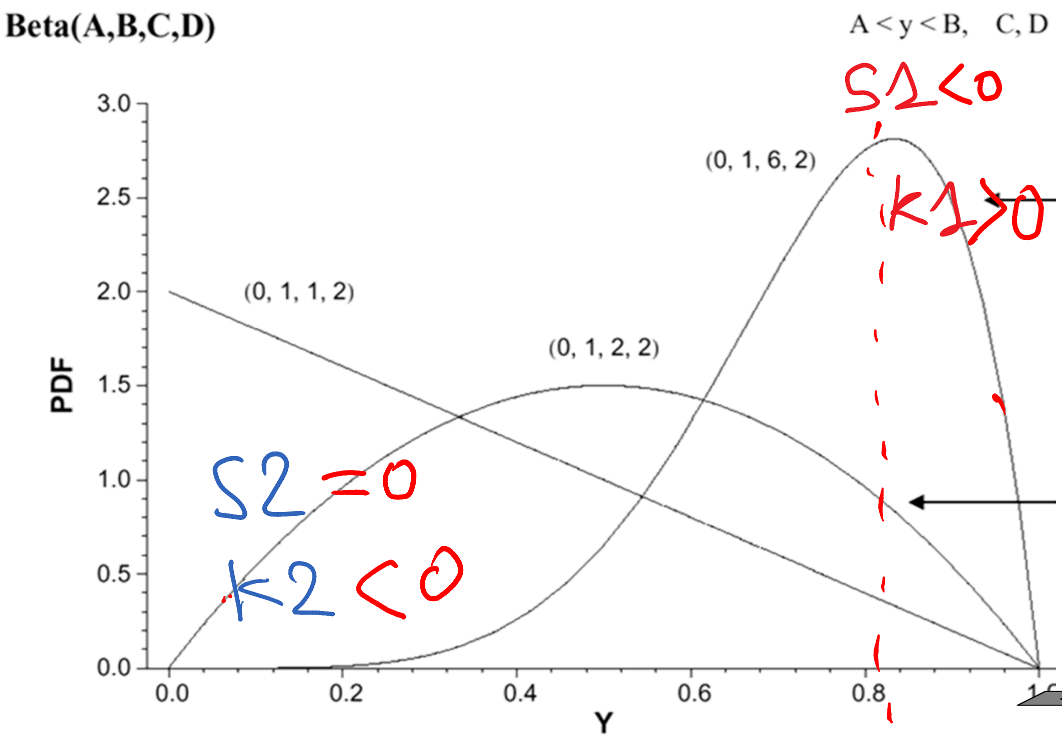


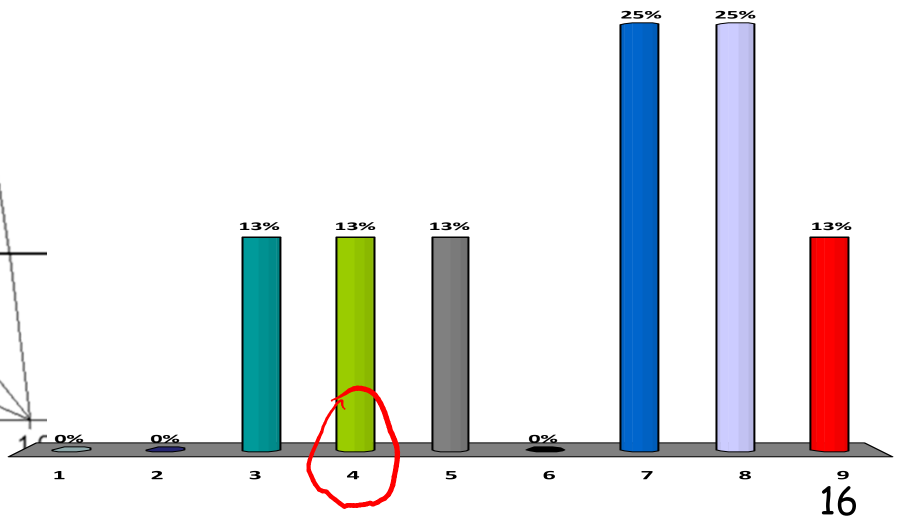
1. $A = 0.5; B = 1; C = 2; D = 3$
2. $A = 3; B = 2; C = 1; D = 0.5$
3. $A = 0.5; B = 2; C = 1; D = 3$
4. $A = 1; B = 2; C = 3; D = 0.5$
5. I don't know



The skewness indices S1, S2 and the Kurtosis indices K1, K2 are…

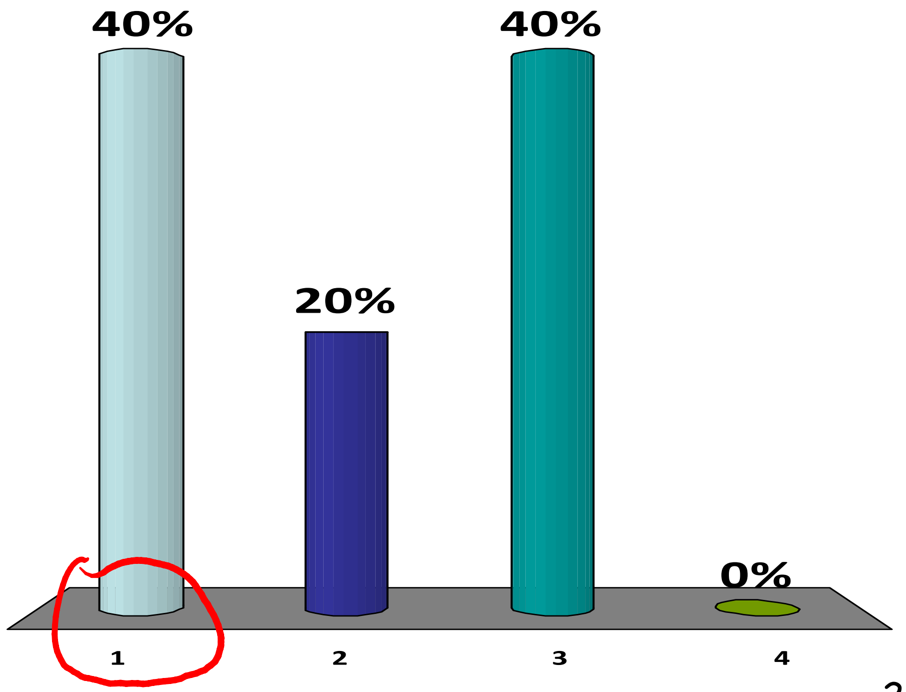
1. S1>0, K1>0, S2>0, K2>0
2. S1<0, K1>0, S2>0, K2<0
3. S1>0, K1>0, S2=0, K2>0
4. S1<0, K1>0, S2=0, K2<0
5. S1>0, K1<0, S2>0, K2>0
6. S1<0, K1<0, S2>0, K2<0
7. S1>0, K1<0, S2=0, K2>0
8. S1>0, K1<0, S2=0, K2<0
9. I don't know





If a positive random variable has a finite mean and is heavy tailed, its variance is infinite

1. True
2. False
3. It depends
4. I don't know

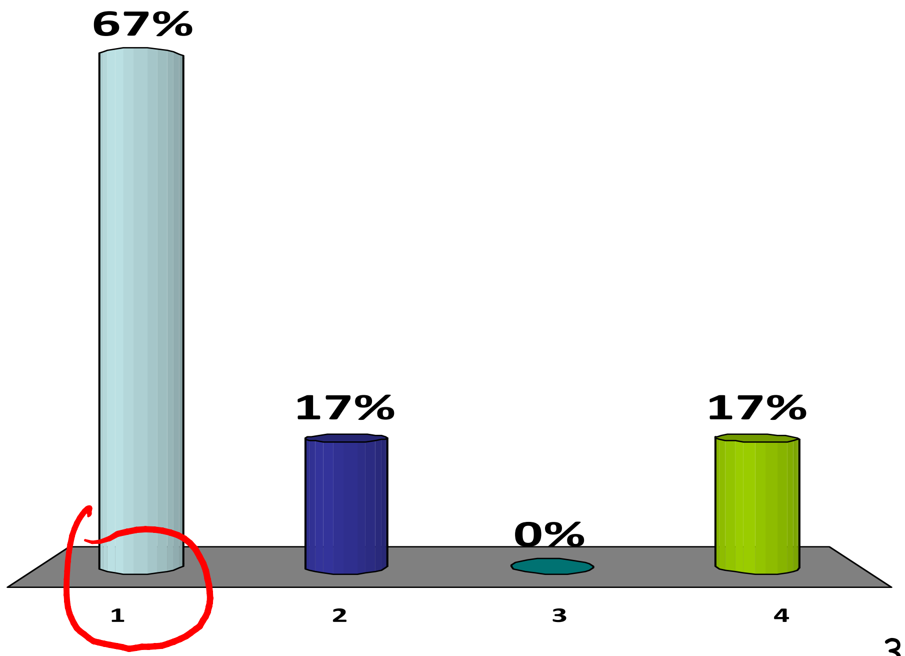


The CCDF of a Pareto distribution follows a power law…

1. True
2. False
3. It depends
4. I don't know

$F(x) = 1 - \frac{1}{x^{\beta}}$

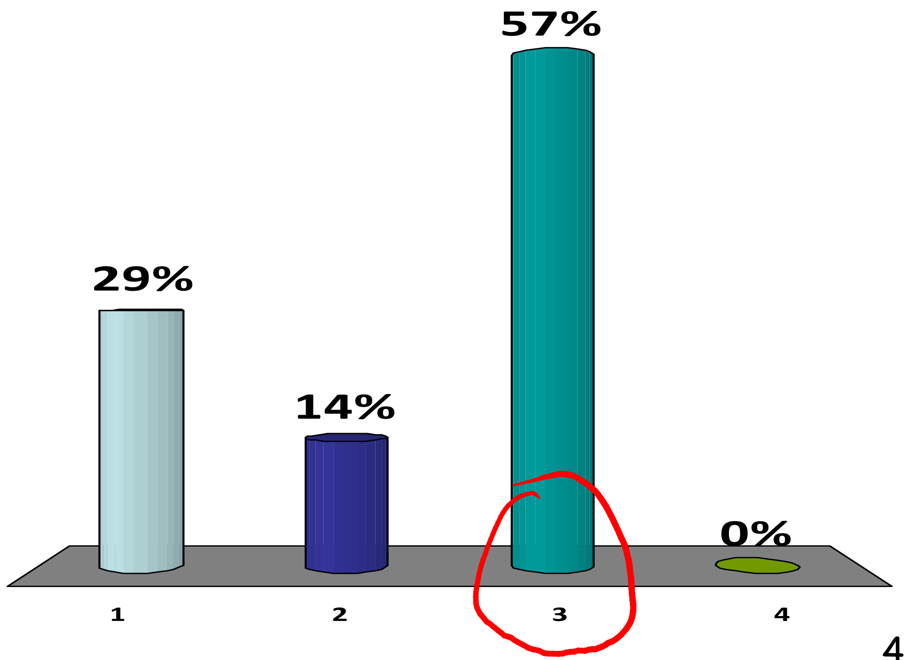
$(x > 1)$



A Pareto distribution is heavy tailed …

1. True
2. False
3. It depends
4. I don't know

p > 2 ⇒ HT



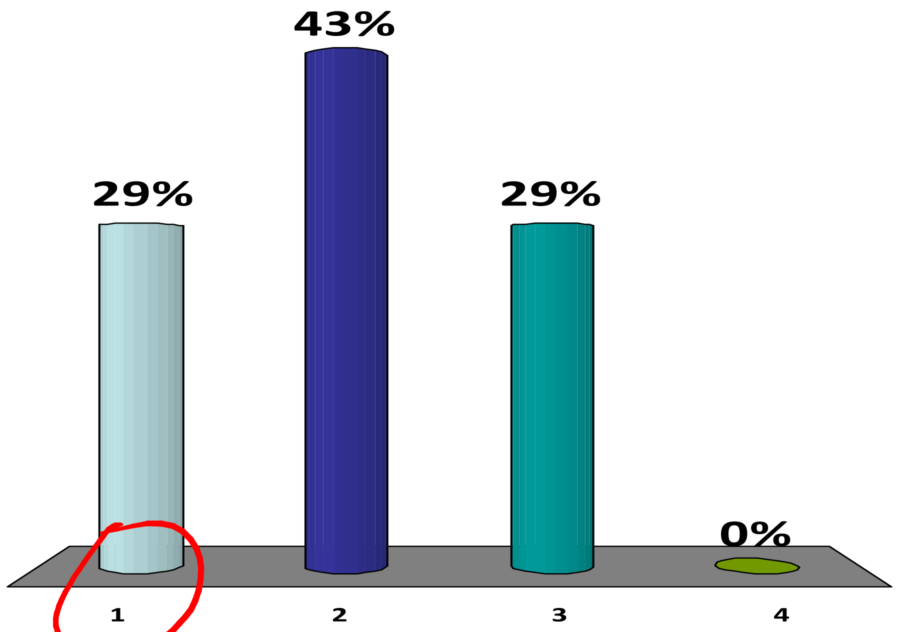
For a Pareto distribution, the hazard rate $\lambda(t)$ is such that

$$\lim_{t \to \infty} \lambda(t) = 0$$

1. True
2. False
3. It depends
4. I don't know

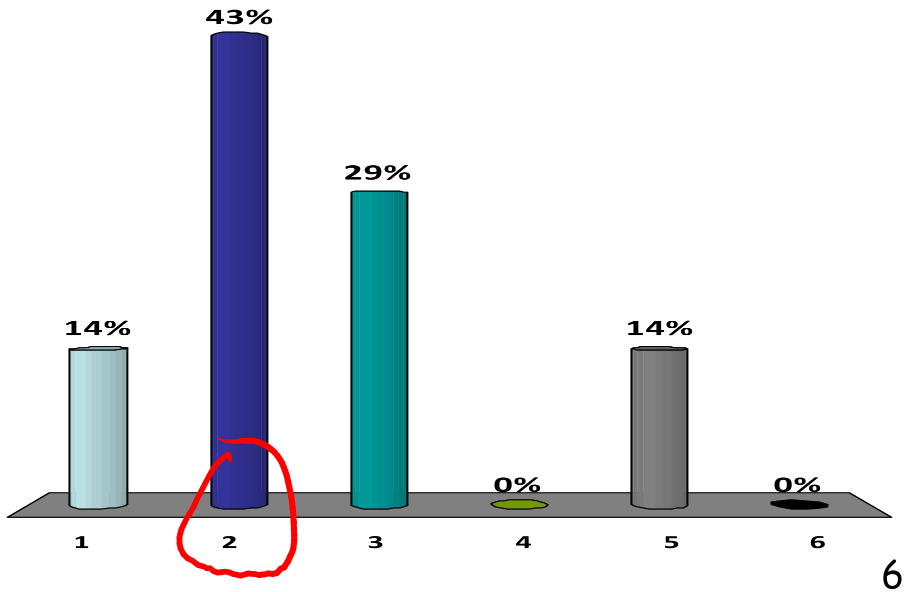
$$\lambda(t) = \frac{f(t)}{1 - F(t)}$$

$$= \frac{p}{x^{p+1}} \cdot \frac{x^p}{1} = \frac{p}{x} \to 0$$



The distribution of the sum of n iid random variables with heavy tail and index $p \cancel{\geq} < 2$, for large n, is approximately…

1. Normal
2. Stable with same index p
3. Stable but not not necessarily with same index p
4. Poisson
5. It depends
6. I don't know



The distribution of the sum of n iid random variables with finite variance, for large n, is approximately…

1. Normal
2. Stable
3. Poisson
4. It depends
5. I don't know