

# For which items do we need to identify the intensity of the load ?

- A Compare Windows 2000 Professional versus Linux.
- B Design a rate control for an internet audio application.
- C Compare various wireless MAC protocols.
- D Say how many servers a video on demand company needs to install.

1. None

2. A

3. B

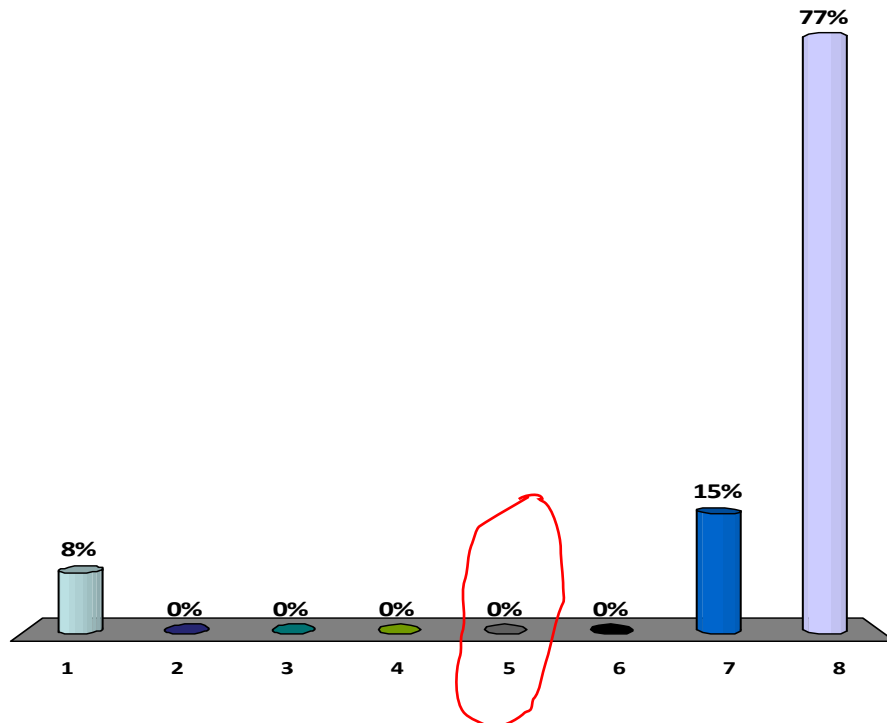
4. C

5. D

6. A and B

7. C and D

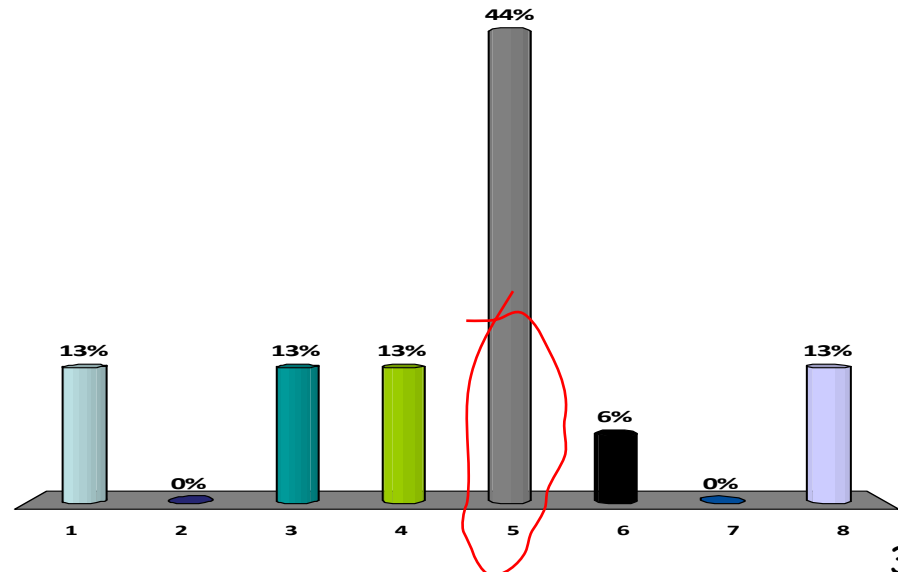
8. All



# Say what is true

1. None
2. A
3. B
4. C
5. D
6. A and C
7. B and D
8. All

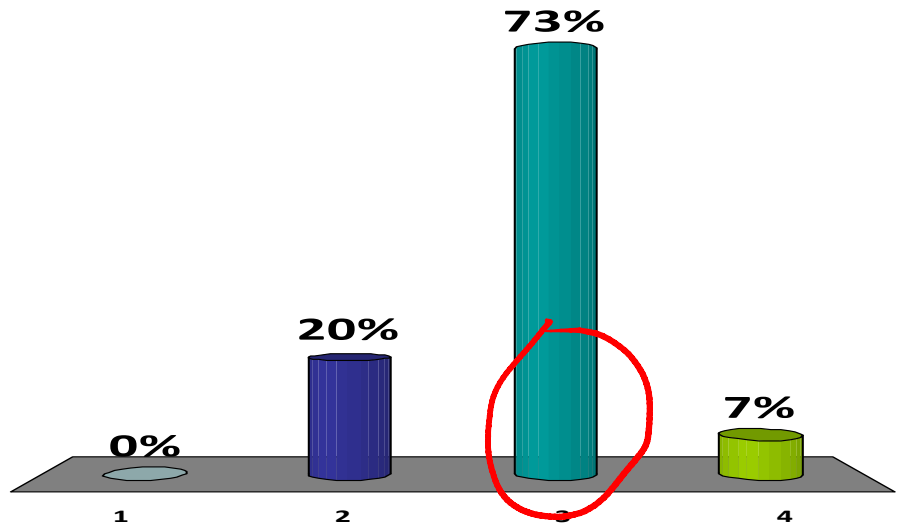
- a «non dominated metric» means
- A. a metric value that is better than or equal to all others
  - B. a metric value that is better than all others
  - C. a metric vector that is better or as good as all others
  - D. a metric vector for which no other vector is better



Items are classified as X1, X2, X3, Y1, Y2 or Y3. Every item has a chance of failing or not. We test all X1 and Y1 items and find that the X1 items have a higher chance of failing than the Y1; idem for X2 vs Y2, X3 vs Y3.

⇒ **an X item has a higher chance of failing than a Y item**

1. True
2. False
3. It depends
4. I don't know



$p_{X1}$  = proba that an item fails given it is X1

$$\left. \begin{array}{l} p_{X1} > p_{Y1} \\ p_{X2} > p_{Y2} \\ p_{X3} > p_{Y3} \end{array} \right\}$$

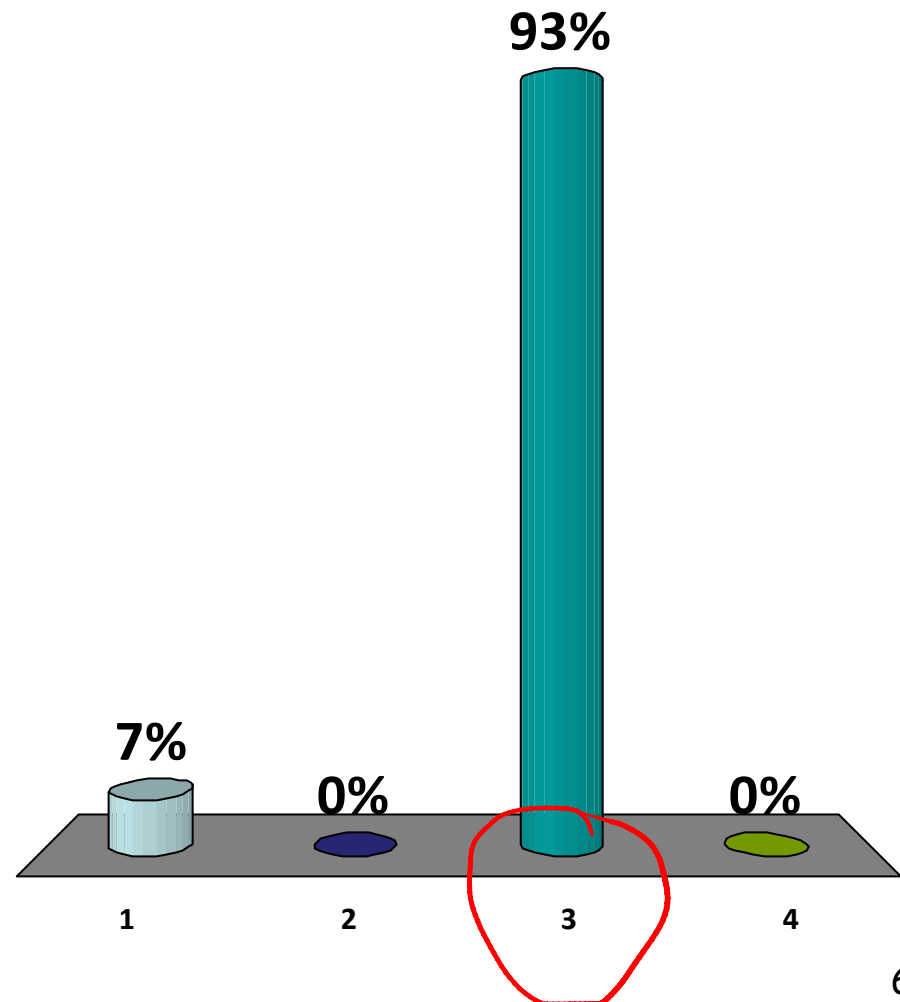
Bayes  $\Rightarrow$

$$\begin{aligned} p_X &= p_{X1}p(1|X) + p_{X2}p(2|X) + p_{X3}p(3|X) \\ p_Y &= p_{Y1}p(1|Y) + p_{Y2}p(2|Y) + p_{Y3}p(3|Y) \end{aligned}$$

- $\rightarrow$  In general  $p(1|X) \neq p(1|Y)$ , the weights are different  
It may be that  $p_X > p_Y$  or  $p_X < p_Y$ , depending on values
- $\rightarrow$  If  $p(i|X) = p(i|Y)$  for all  $i$  (randomized experiments) then  
 $p_X > p_Y$

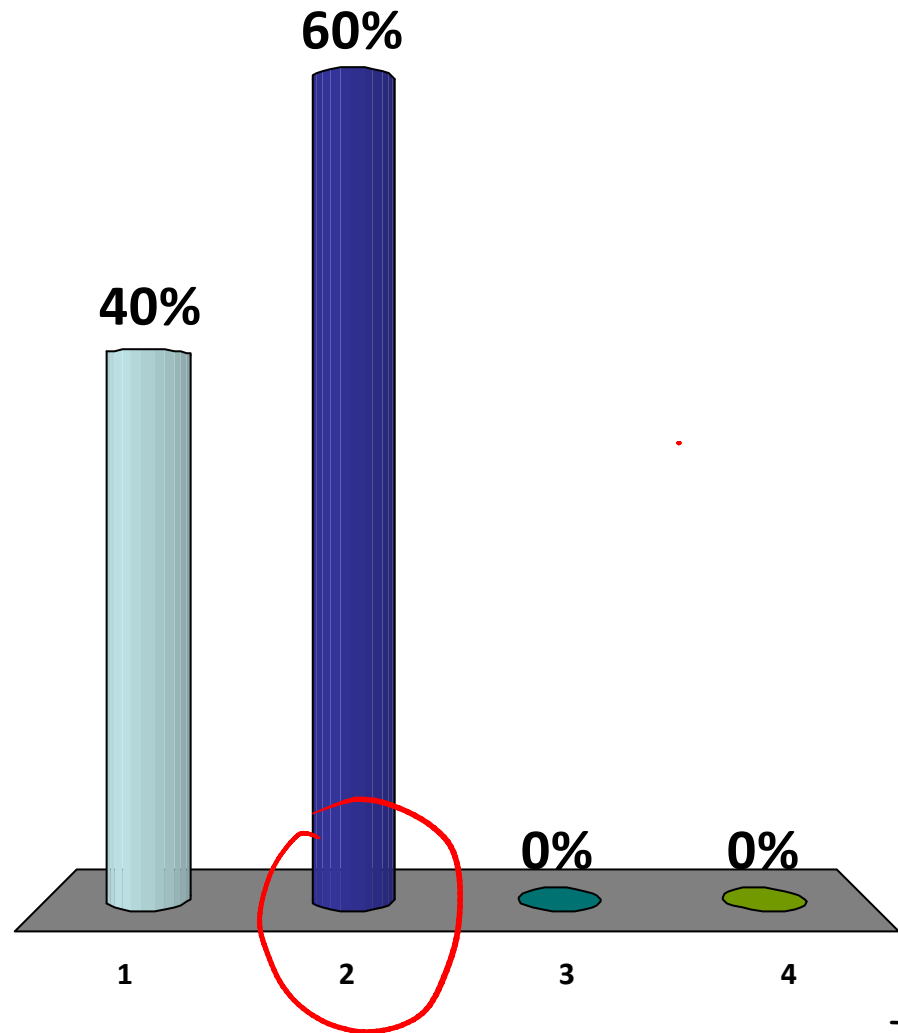
# The «scientific method» means

1. Carefully screen all experimental conditions
2. Beware of hidden factors
3. Do not draw a conclusion until you have exhausted all attempts to invalidate it
4. I do not know



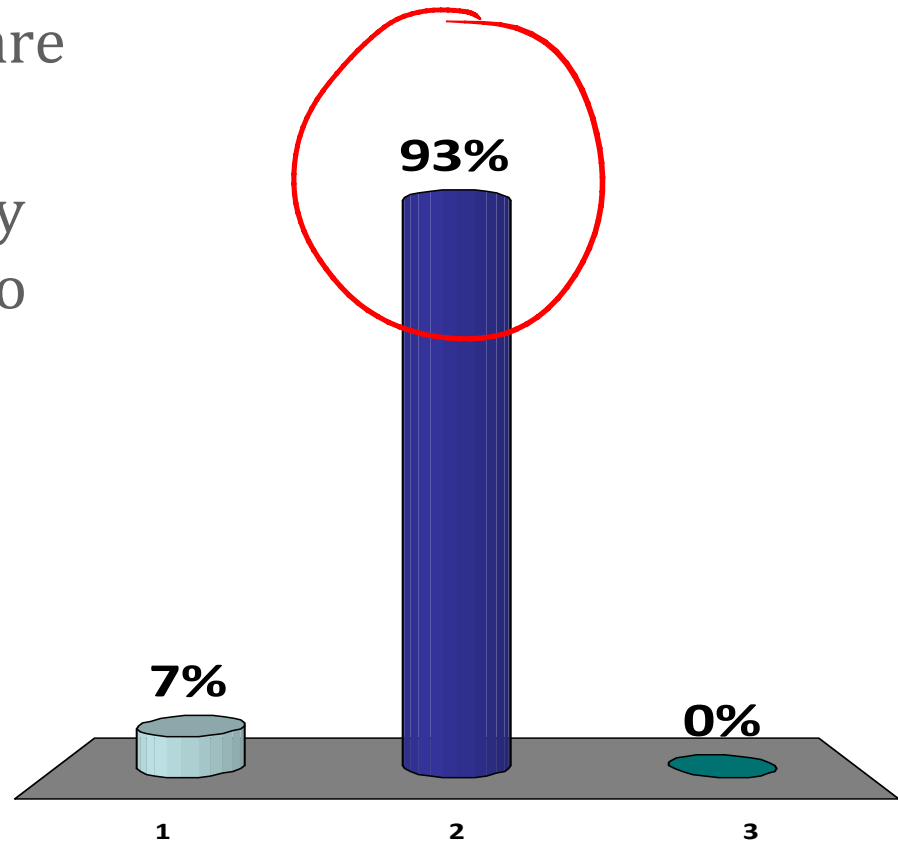
# A nuisance factor is

1. An unanticipated experimental condition that corrupts the results
2. A condition in the system that affects the performance but that we are not interested in
3. An unpleasant part of the performance evaluation
4. I do not know



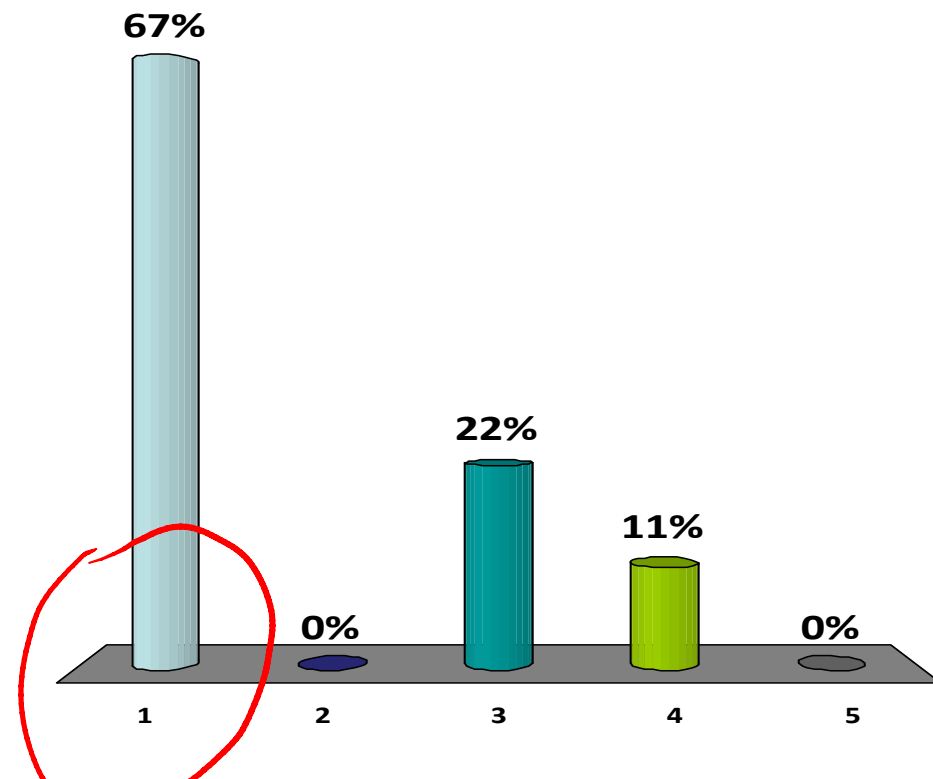
# Joe increases the number of gateways and throughput decreases.

1. This is impossible in theory, it must be due to a software bug
2. This may happen in a fully functional system, with no software bug
3. I do not know



We increase the number of gateways and throughput decreases. Which pattern can explain this?

1. Latent congestion collapse
2. Bottleneck
3. Competition side effect
4. Congestion collapse
5. I don't know



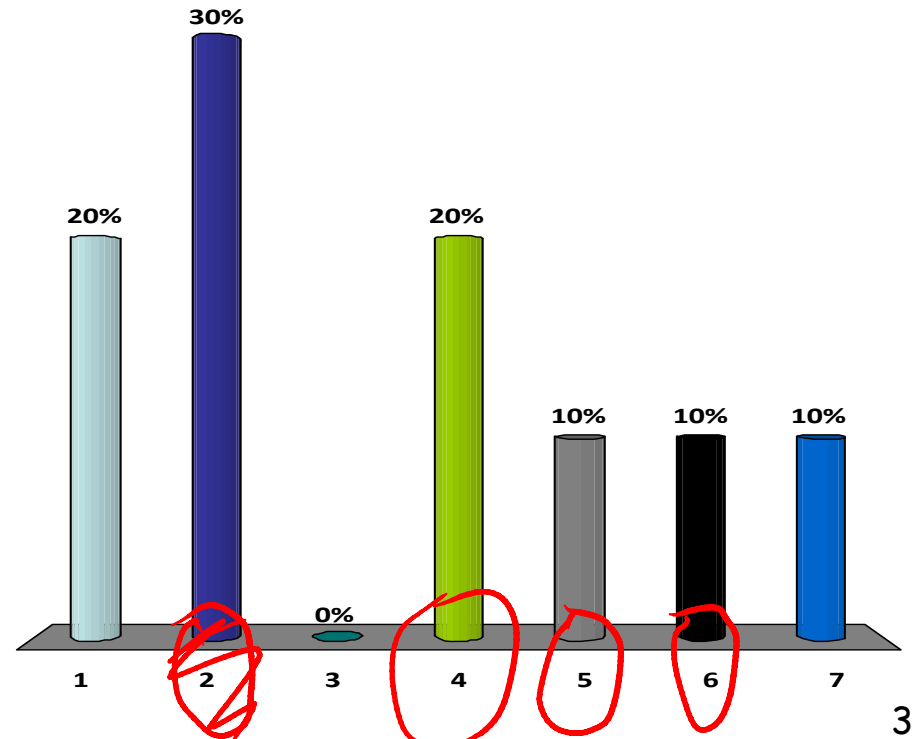


We do 10 independent simulations to measure a response time and obtain (in ms):

- 1.01
- 1.02
- 0.98
- 1.00
- 1.02
- 100.00
- 1.01
- 0.99
- 1.00
- 1.01

Give a 95% confidence interval for the response time.

- 1. [ 0.98 ; 100 ]
- 2. [ 0.98 ; 1.02 ]
- 3. [ 0.99 ; 100.00 ]
- 4. [ 0.99 ; 1.02 ]
- 5. [-7.5 29.3]
- 6. [-8.5 30.3]
- 7. I don't know



## Solution

- Method 1: 95% confidence interval for median

$$[x_{(2)}, x_{(9)}] = [0.99 ; 1.02]$$

$$\text{Median is } \frac{x_{(5)} + x_{(6)}}{2} = 1.01$$

answer 4 is correct

- Method 2: 95% confidence interval for mean is  $m \pm 1.96 \frac{s}{\sqrt{n}}$

$$\text{mean } m = 10.9$$

$$s = 31.3 \text{ with } \frac{1}{n} \text{ formula}$$

$$\sigma = 29.7 \text{ with } \frac{1}{n-1} \text{ formula}$$

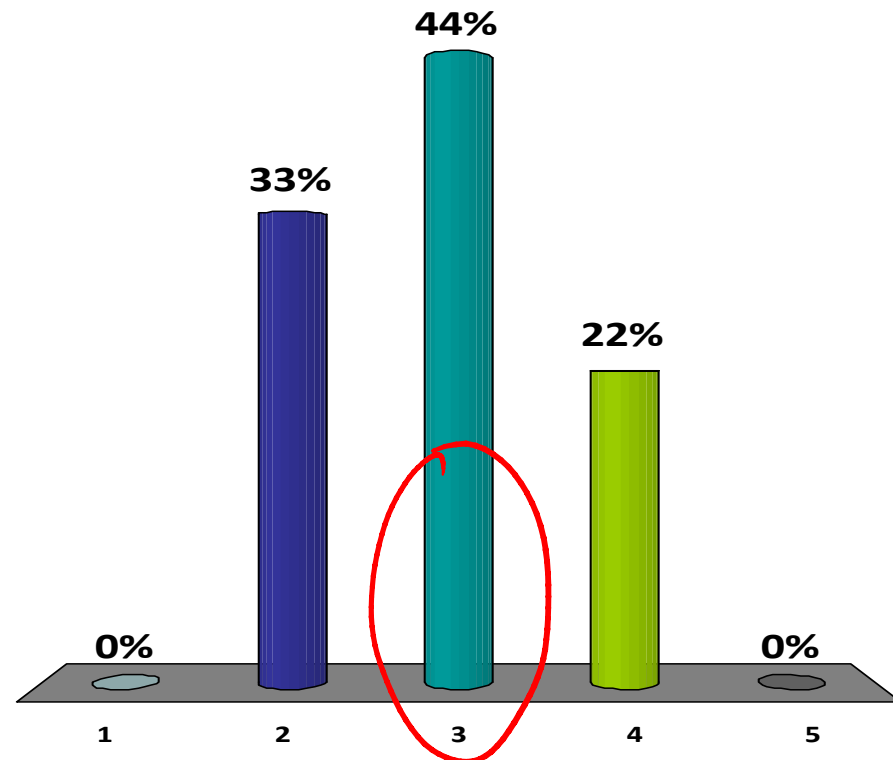
$$\text{CI} = [-8.5 \ 30.3] \text{ or } [-7.5 \ 29.3]$$

answers 5 and 6 are correct

- Method 1 is more robust

We have tested a system for errors and found 0 error in 37 runs. Give a confidence interval for the probability of error.

1. [0% ; 2.7%] ←
2. [0% ; 5.3%] ←
3. [0% ; 9.5%]
4. [0% ; 19.5%]
5. I don't know



## Solution

CI is  $[0 ; p_0(n)]$

For  $\gamma = 0.95$ , Eq.(2.28) gives  $p_0(n) \approx \frac{3.689}{n}$  and this is accurate with less than 10% relative error for  $n \geq 20$  already.

$$p_0(n) = 1 - \left( \frac{1 - \gamma}{2} \right)^{\frac{1}{n}}$$

$$p_0(37) \approx \frac{3.689}{37} \approx 10\%$$

Which algorithm is a correct bootstrap-computation of a 95%-confidence-interval for Jain's fairness index ?

```

Algorithm A
for  $n = 1:37$ 
  for  $r = 1:99$ 
    draw one sample  $y_{n,r}$  from  $S$ 
  end
end
for  $r = 1:99$ 
  compute Jain's fairness index  $f_r$  of  $y_{1,r}, \dots, y_{37,r}$ 
end
CI =  $[f_{(5)}, f_{(95)}]$ 
  
```

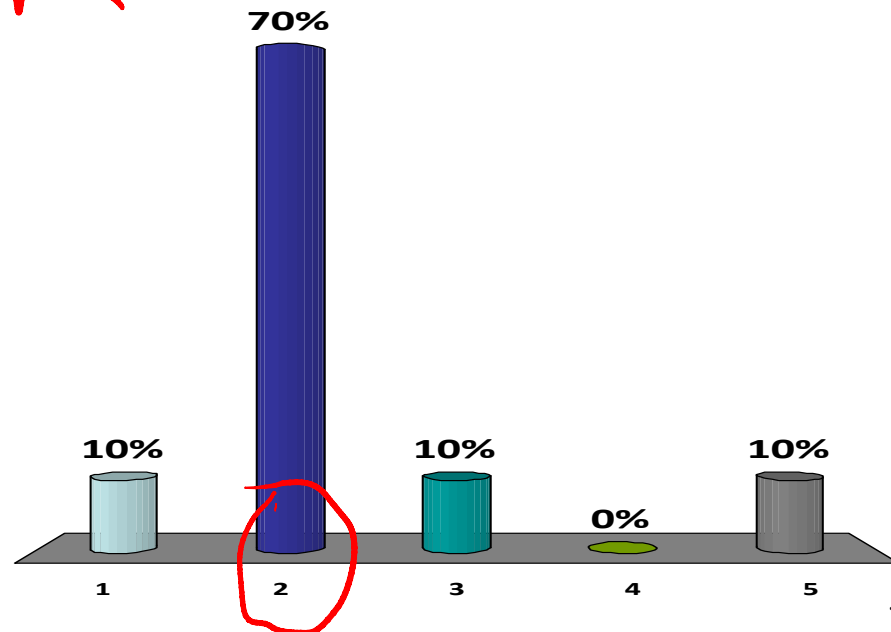
OK

```

Algorithm B
for  $r = 1:99$ 
  draw one random permutation  $\sigma_r$  of  $\{1, \dots, 37\}$ 
end
for  $r = 1:99$ 
  compute Jain's fairness index  $f_r$  of  $x_{\sigma_r(1)}, \dots, x_{\sigma_r(37)}$ 
end
CI =  $[f_{(5)}, f_{(95)}]$ 
  
```

(we are given a set of 37 independent results  $S = \{x_1, \dots, x_{37}\}$ )

1. None
2. A
3. B
4. Both
5. I don't know



## Solution

```
for  $r = 1:99$ 
  for  $n = 1:37$ 
    draw one sample  $y_{n,r}$  from  $S$ 
  end
end
for  $r = 1:99$ 
  compute Jain's fairness index
   $f_r$  of  $y_{1,r}, \dots, y_{37,r}$ 
end
CI = [ $f_{(5)}, f_{(95)}$ ]
```

The textbook implementation of the bootstrap consists in making  $R$  replay experiments by re-sampling with replacement from the data.

This is the same as Algorithm A

We have two independent measurements

$$X_1 = 7.4 \text{ and}$$

$$X_2 = 8.0. \text{ Let}$$

$$L = \min(X_1, X_2) \text{ and}$$

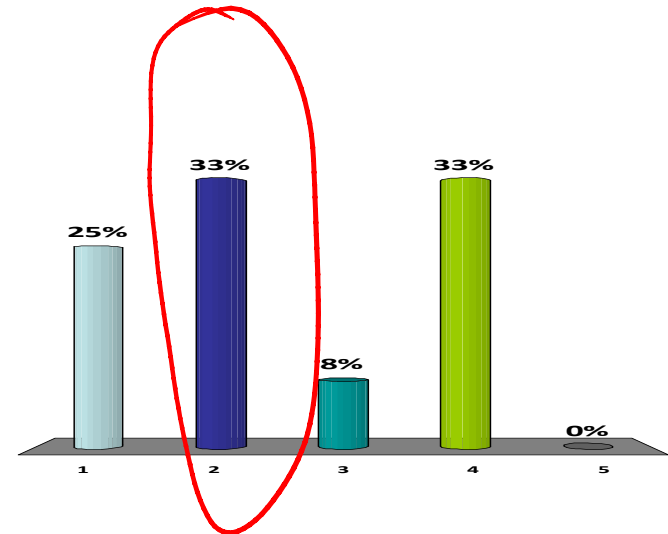
$$U = \max(X_1, X_2).$$

A.  
The probability of the event  $\{L \leq \theta \leq U\}$  is 0.5

B.  
The probability of the event  $\{7.4 \leq \theta \leq 8.0\}$  is 0.5

Say which statement is correct about the median  $\theta$  of the distribution of  $X_1$  and  $X_2$ .

1. None
2. A
3. B
4. Both
5. I don't know



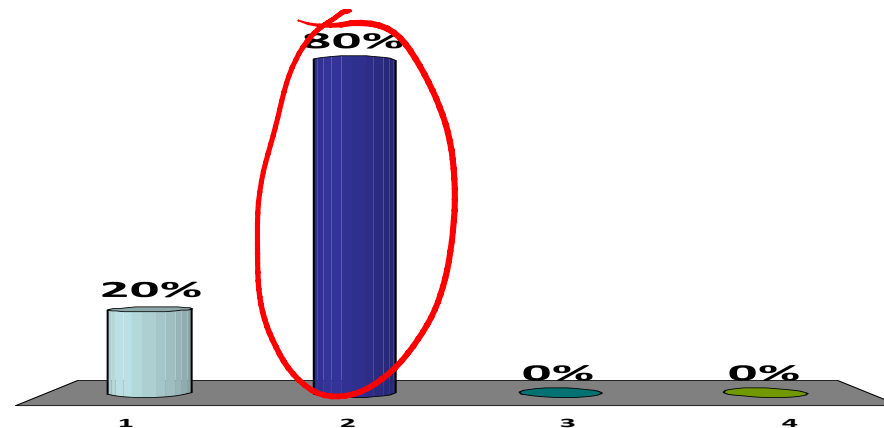
## Solution

1. Imagine that the distribution is continuous with a density. Thus the probability that one measurement is  $\geq \theta$  is 0.5 and the probability that one measurement is  $\leq \theta$  is 0.5. The event in statement A means: one of the measurements is  $\geq \theta$ , the other is  $\leq \theta$ . This happens with proba 0.5. Statement A is correct.
2. Statement B does not have a meaning, there is no probability assigned to events expressed in terms of the parameter  $\theta$ .



## We expect...

1. ... a 95%-confidence interval to be wider than a 99%-confidence interval .
2. ... a 95%-confidence interval to be narrower than a 99%-confidence interval.
3. It depends on the data
4. I don't know



## Solution

If we want to be more certain, we need the interval to be wider.

Check this with the confidence interval for the median ( $n = 37$ ):

$$95\% : [x_{(13)}, x_{(25)}]$$

$$99\% : [x_{(11)}, x_{(27)}]$$

For the mean

$$95\%: m \pm 1.96 \frac{s}{\sqrt{n}}$$

$$99\%: m \pm 2.58 \frac{s}{\sqrt{n}}$$

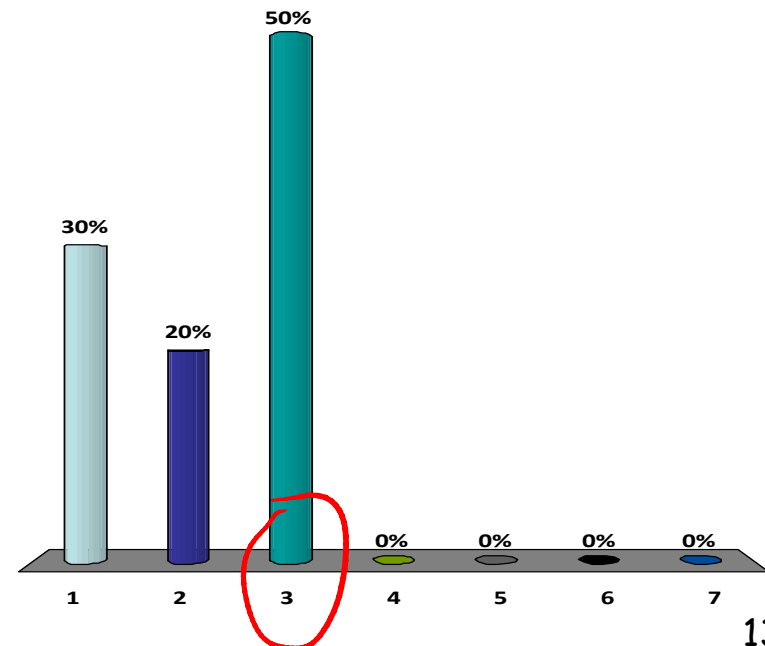
A data set  $\{x_i, i = 1 \dots 39\}$  contains 39 independent values. Which interval is a prediction interval at level 95% ?

A:  $[x_{(13)}, x_{(27)}]$

B:  $[x_{(1)}, x_{(39)}]$

C:  $m \pm 1.96 \frac{s}{\sqrt{37}}$  where  $m$  is the mean and  $s$  is the standard deviation

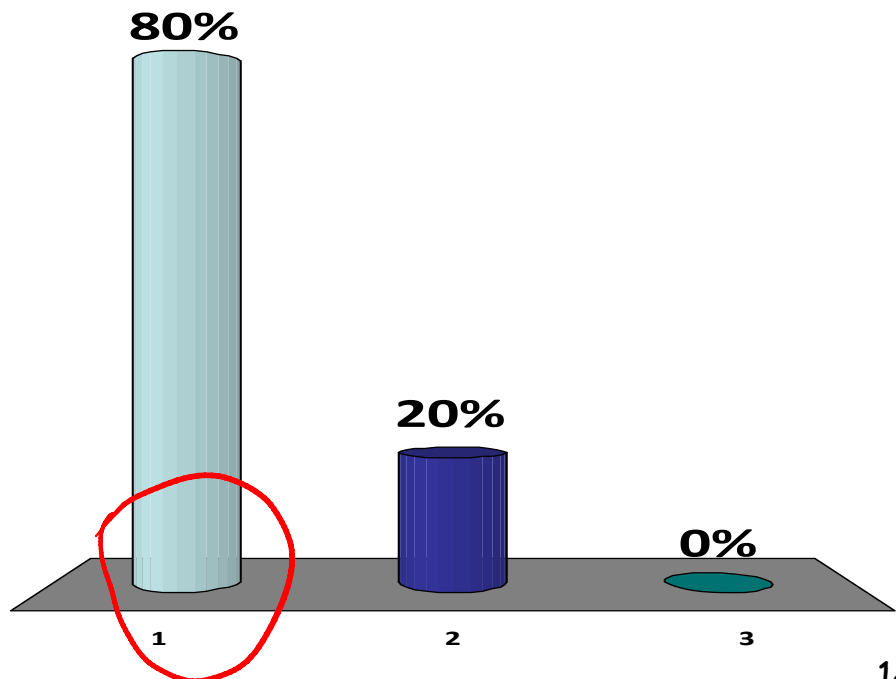
1. None
2. A
3. B
4. C
5. A and C
6. B and C
7. I don't know



A data set  $x_i$  is such that  $y_i = \log(x_i)$  looks normal. A 95%-prediction interval for  $y_i$  is  $[L, U]$ .

Is it true that a 95%- prediction interval for  $x_i$  is  $[e^L, e^U]$  ?

1. True
2. False
3. I don't know



## Solution

$\rightarrow P\{L \leq Y_{n+1} \leq U\} \geq 95\%$   
thus

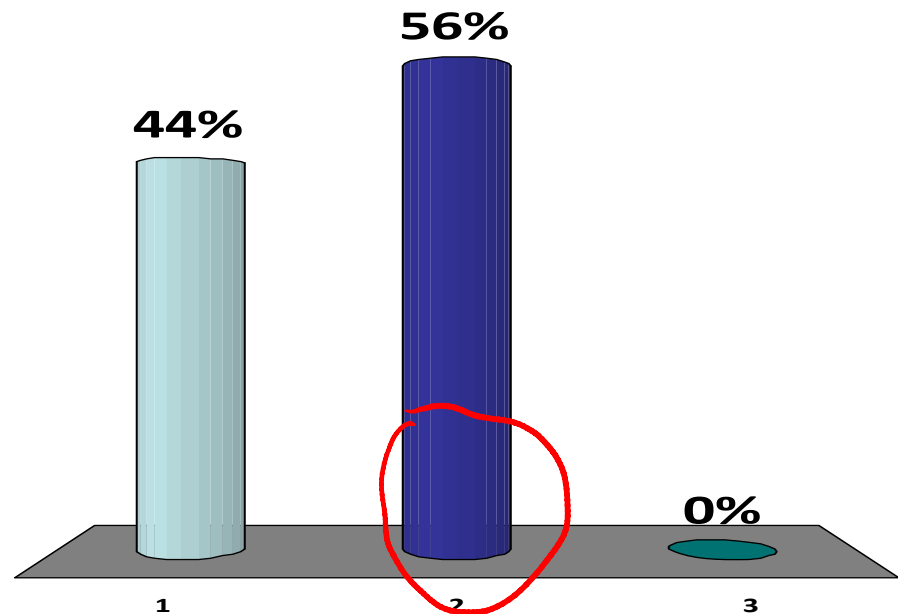
$\rightarrow P\{e^L \leq e^{Y_{n+1}} \leq e^U\} \geq 95\%$   
 $P\{e^L \leq X_{n+1} \leq e^U\} \geq 95\%$

True

A data set  $x_i$  is such that  $y_i = \log(x_i)$  looks normal. A 95%-confidence interval for the mean of  $y_i$  is  $[L, U]$ .

Is it true that a 95%-confidence interval <sup>for</sup> the mean of  $x_i$  is  $[e^L, e^U]$ ?

1. True
2. False
3. I don't know



## Solution

$$g_X = \left( \prod x_i \right)^{1/n}$$

False:

$$\underline{X_i = e^{Y_i}, \text{ but } m_X \neq e^{m_Y} \text{ in general}}$$

It is the geometric mean  $g_X$

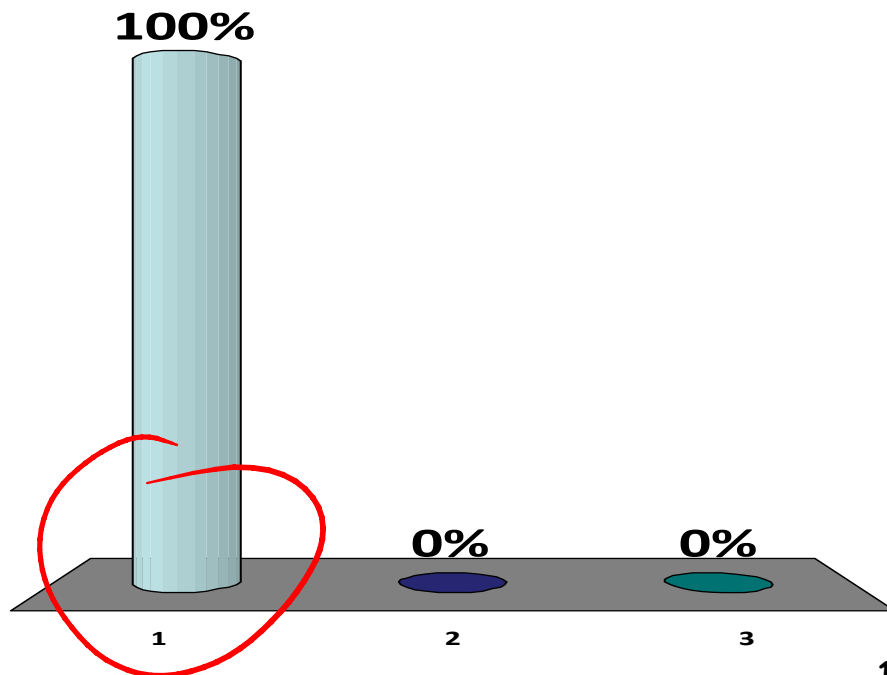
$$\underline{g_X = e^{m_Y}}$$

$[e^L ; e^U]$  is a confidence interval for the geometric mean  $g_X$

A data set  $x_i$  is such that  $y_i = \log(x_i)$  looks normal. A 95%-confidence interval for the median of  $y_i$  is  $[L, U]$ .

Is it true that a 95%- confidence interval the median of  $x_i$  is  $[e^L, e^U]$  ?

1. True
2. False
3. I don't know





## Solution

A confidence interval for the median of  $y_i$  is given by  $L = y_{(j)}$ ,  $U = y_{(k)}$

where  $j$  and  $k$  are given by the table and depend on  $n$

A confidence interval for the median of  $x_i$  is given by  $L_X = x_{(j)}$ ,  $U_X = x_{(k)}$

where  $j$  and  $k$  are the same as for  $y_i$

$x_{(j)} = e^{y_{(j)}}$  and  $x_{(k)} = e^{y_{(k)}}$  because exp is monotonic  $\uparrow$

$[e^L ; e^U]$  is a confidence interval for the median of  $x_i$

True

A set of measurements is positively correlated. We compute a confidence interval for the median as if it were iid. The true confidence interval is ...

1. Larger
2. Smaller
3. It depends on the data
4. I don't know

